LINEAR PROGRAMMING EXAMPLES AND SOLUTIONS

This file contains <u>four</u> Linear Programming problems examples. Each problem is first expressed in **mathematical notation**; next the models are expressed in **GAMS notation**; and finally the model is solved and the results are found in the a **GAMS LIST** file. We will go over the problems in class and interpret the results. You will then write and execute **GAMS** programs to solve the problems assigned to you.

GAMS program is written in **TEXT** format. You can either write your model in GAMS notation in the GAMS Editor window or use **WORD** or **NOTEPAD** to type the GAMS commands for the problem you are working on and save the file as a **TEXT** file and then paste it into GAMS Editor. Save the files under names which will help you remember as to what the file contains. For example **PROB1** if the file pertains to Problem 1.

When writing your **GAMS** code, never use **TABS** and always leave the first position of each line blank (it is reserved for the special characters). For example, a "*" character in the first position will allow you to write notes or comments that the program will ignore.

When you are ready to solve the problem by executing the GAMS program, click on the "RUN" icon. If you made no errors, **GAMS** will solve the problem and it will save the results in a list file with the same name but with an extension "**LST**." For example **PROB1.LST**. If you made errors, GAMS will indicate it on the monitor. Errors are found by looking at lines the start with **** in the **LST** file. There will be a **$** sign right under the place where your error has occurred followed by a number. At the end of the **LST** file, the error(s) number will be listed followed by an explanation. Once you know your error(s), go back into your **GAMS** command file and make corrections. Save the file and run the program again.

You can download a User's Guide and a free student copy of **GAMS Window** version from the web site **http://palette.ecn.purdue.edu/~rardin/gams/notes.html** and download **gidev091.exe**.

EXAMPLE 1:

A clothes manufacturer makes products $\mathbf{Q_1}$: Pants and $\mathbf{Q_2}$: Jackets. The net profit from Pants p_1=\$5 and from jackets p_2=\$10. The firm is facing the following time constraints on the production processes:

Cutting	$0.5Q_1 + 2.5Q_2 = 100$ hours
Sawing	$1.0Q_1 + 0.5Q_2 = 70$ hours
Packaging	$1.0Q_1 + 1.0Q_2 = 80$ hours

Find the profit maximizing levels of Q_1 and Q_2.

EXAMPLE 1: **Model in Mathematical Notation**

Maximize $\Pi = \sum_{i=1}^{2} p_i * Q_i$

Subject to: $\sum_{i=1}^{2} a_{ij} * Q_i \leq b_j$ i= 1,2 and j = 1,2,3

Where: p_i is the net profit and Q_i is the output of good i, respectively
a_{ij} is amount of resource j used per unit of output of good i
b_j is the total amount of resource j available.

EXAMPLE 1: **Model in GAMS Notation**

```
$TITLE DR. KONYAR'S LINEAR PROGRAMMING EXAMPLE 1
$OFFSYMLIST OFFSYMXREF
 OPTION NLP = MINOS5

 SETS
   I GOODS   /JACK, PANT /
   J PRODUCTION PROCESSES  /CUT, SAW, PACK/;

*Data
  PARAMETERS
   NETPROF(I) NET PROFIT PER UNIT
  / JACK  5
    PANT 10 /

   PRDLEVEL(J)  PRODUCTION CONSTRAINT LEVELS
  / CUT   100, SAW    70, PACK   80 /;

  TABLE
   USEPARAM(J,I) PRODUCTION PROCESS USE PARAMETERS

          JACK PANT      (Note: When creating data in a TABLE format, at least
  CUT     .5   2.5        one part of the number must be vertically aligned
  SAW      1   .5         with the column label under which it belongs)
  PACK     1    1 ;

 VARIABLES
   Q(I) OUTPUT LEVEL
   PROF TOTAL PROFIT;

 POSITIVE  VARIABLE Q;   (Note: This is akin to the non-negativity constraint)

 EQUATIONS
  OBJFUNC   OBJECTIVE FUNCTION
  CONST(J)  CONSTRAINS;

 OBJFUNC .. PROF =E= SUM(I, NETPROF(I)*Q(I));
 CONST(J) ..  SUM(I, Q(I) * USEPARAM(J,I)) =L= PRDLEVEL(J);

 MODEL PROFIT /OBJFUNC,CONST/
 SOLVE PROFIT USING LP MAXIMIZING PROF;
```

EXAMPLE 1: **GAMS LIST FILE**

```
GAMS 2.25.087  386/486 DOS                02/25/98 14:41:35  PAGE      1
DR. KONYAR'S  LINEAR PROGRAMMING EXAMPLE

COMPILATION TIME     =        0.050 SECONDS        VERID MW2-25-087
Equation Listing    SOLVE PROFIT USING LP FROM LINE 53


---- OBJFUNC     =E=  OBJECTIVE FUNCTION


OBJFUNC..  - 5*Q(JACK) - 10*Q(PANT) + PROF =E= 0 ; (LHS = 0)


---- CONST       =L=  CONSTRAINS

CONST(CUT)..  0.5*Q(JACK) + 2.5*Q(PANT) =L= 100 ; (LHS = 0)

CONST(SAW)..  Q(JACK) + 0.5*Q(PANT) =L= 70 ; (LHS = 0)

CONST(PACK)..  Q(JACK) + Q(PANT) =L= 80 ; (LHS = 0)

MODEL STATISTICS

BLOCKS OF EQUATIONS       2     SINGLE EQUATIONS        4
BLOCKS OF VARIABLES       2     SINGLE VARIABLES        3
NON ZERO ELEMENTS         9

GENERATION TIME      =        0.050 SECONDS

EXECUTION TIME       =        0.170 SECONDS        VERID MW2-25-087
Solution Report     SOLVE PROFIT USING LP FROM LINE 53

               S O L V E      S U M M A R Y

     MODEL   PROFIT              OBJECTIVE  PROF
     TYPE    LP                  DIRECTION  MAXIMIZE
     SOLVER  MINOS5              FROM LINE  53

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE              550.0000

 RESOURCE USAGE, LIMIT          0.270     5400.000
 ITERATION COUNT, LIMIT         2        10000

     M I N O S    5.3    (Nov 1990)         Ver: 225-386-02
     = = = = =

EXIT -- OPTIMAL SOLUTION FOUND



                       LOWER     LEVEL     UPPER    MARGINAL

---- EQU OBJFUNC         .         .         .        1.000
  OBJFUNC     OBJECTIVE FUNCTION


---- EQU CONST       CONSTRAINS

        LOWER     LEVEL     UPPER    MARGINAL

CUT      -INF    100.000   100.000     2.500
SAW      -INF     65.000    70.000      .
PACK     -INF     80.000    80.000     3.750
```

Note: Marginal value is same as the <u>opportunity cost</u> of a scarce, i.e., binding resource. It is also know as <u>dual value</u>, <u>shadow value</u>, <u>imputed value</u>, or <u>accounting value</u>. It measures the amount by which the objective function value would change if the resource constraint was increased by one unit. For example, if the cutting constraint was to be increased by 1 hour then the profits will increase by $2.50. Which means the cutting capacity has a scarcity value of $2.50.

```
---- VAR Q           OUTPUT LEVEL

     LOWER     LEVEL     UPPER    MARGINAL

JACK   .       50.000     +INF       .
PANT   .       30.000     +INF       .

Solution Report     SOLVE PROFIT USING LP FROM LINE 53

                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR PROF           -INF    550.000     +INF       .

  PROF        TOTAL PROFIT


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED


EXECUTION TIME       =        0.160 SECONDS        VERID MW2-25-087


USER: U.S. Department of Agriculture                 G950224:1314CR-MW2
      Washington, DC

**** FILE SUMMARY

INPUT      C:\COURSE\E480\EXP1.GMS
OUTPUT     C:\COURSE\E480\EXP1.LST
```

EXAMPLE 2

Constrained Cost Minimization

Linear programming is an optimization technique for finding the set of values for the decision variables that maximizes or minimizes an objective function subject to one or more constraints. In the preceding section, a profit function was maximized subject to resource constraints in the form of a limited number of available machine hours. In this section the solution to a minimization problem is outlined. The approach is analogous to that used in the maximization problem.

Structuring the Problem

The example used here comes from agribusiness, where linear programming models have been widely used. Consider a producer of milk whose objective is to feed the milk cows adequately but to do so at minimum cost. Suppose that an adequate feed ration consists of a minimum of 40 units of protein, 60 units of calcium, and 60 units of carbohydrates.

The manager must determine how much of two feeds, *A* and *B*, to use. One ton of feed *A* contains one unit of protein, three units of calcium, and one unit of carbohydrates. One ton of feed *B* contains one unit of protein, one unit of calcium, and six units of carbohydrates. Let the price of feed *A* be $100 per ton and the price of feed *B* be $200 per ton. These basic data are summarized in Table 8-4.

The problem is to find the quantities of the two feeds, X_A and X_B (the decision variables), to be purchased so that the feed cost (*C*) will be minimized. Thus the problem is to minimize the objective function,

$$C = 100X_A + 200X_B$$

subject to the following minimum nutrition requirements or constraints:

$$1X_A + 1X_B \geq 40 \quad \text{(protein constraint)}$$
$$3X_A + 1X_B \geq 60 \quad \text{(calcium constraint)}$$
$$1X_A + 6X_B \geq 60 \quad \text{(carbohydrate constraint)}$$

and the usual requirement that all decision variables be nonnegative:

$$X_A \geq 0 \qquad X_B \geq 0$$

Summary of Data for the Cost-Minimization Problem

	Feed		
	A	*B*	
Price per ton	$100	$200	
	Units of Nutrients Per Ton of Feed		
	A	*B*	*Minimum Units Required Per Period*
Protein	1	1	40
Calcium	3	1	60
Carbohydrates	1	6	60

EXAMPLE 2: **Model in Mathematical Notation**

Minimize $C = \sum_{i=1}^{2} c_i * Q_i$

Subject to: $\sum_{i=1}^{2} a_{ij} * Q_i \geq b_j$ i= 1,2 and j = 1,2,3

Where: c_i is the per unit cost of and Q_i is the amount of feed i, respectively
a_{ij} is amount of nutrient j provided by a unit of feed i
b_j is the minimum amount of nutrient j needed per cattle

EXAMPLE 2: **Model in GAMS Notation**

```
$TITLE PETERSEN & LEWIS CHAPTER 8, Page 282
* Constrained Cost Minimization: A farmer is trying to minimize feed cost of feeding a
* cow, subject to minimum nutritional requirements of the cow.

$OFFSYMLIST OFFSYMXREF
 OPTION NLP = MINOS5

 SETS
   I FEEDS   / A, B /
   J NUTRIENTS   / PROTEIN, CALCIUM, CARBO /

*Data
  PARAMETERS
   COST(I) FEED PRICES
  / A  100
    B  200 /

   MINREQ(J)  MINIMUM FEED REQUIREMENTS
  / PROTEIN  40
    CALCIUM  60
    CARBO    60 /;

  TABLE
   NUTRI(J,I) UNITS OF NUTRIENTS PER TON
              A         B
  PROTEIN     1         1
  CALCIUM     3         1
  CARBO       1         6  ;

 VARIABLES
   EAT(I)  FEED FEEDING LEVELS
   TOTCOST TOTAL COST;

 POSITIVE  VARIABLE EAT;

 EQUATIONS
  OBJFUNC   OBJECTIVE FUNCTION
  CONST(J)  CONSTRAINS;

 OBJFUNC ..   TOTCOST =E= SUM(I, COST(I)*EAT(I));
 CONST(J) ..  SUM(I, EAT(I)*NUTRI(J,I)) =G= MINREQ(J);

 MODEL MINCOST /OBJFUNC,CONST/
 SOLVE MINCOST USING LP MINIMIZING TOTCOST;
```

EXAMPLE 2: **GAMS LIST FILE**

```
GAMS 2.25.087  386/486 DOS                    02/25/98 14:41:42  PAGE 1
PETERSEN & LEWIS CHAPTER 8, Page 282

COMPILATION TIME     =        0.050

Equation Listing    SOLVE MINCOST USING LP FROM LINE 57

---- OBJFUNC     =E=  OBJECTIVE FUNCTION

OBJFUNC..  - 100*EAT(A) - 200*EAT(B) + TOTCOST =E= 0 ; (LHS = 0)

---- CONST       =G=  CONSTRAINS

CONST(PROTEIN)..  EAT(A) + EAT(B) =G= 40 ; (LHS = 0 ***)

CONST(CALCIUM)..  3*EAT(A) + EAT(B) =G= 60 ; (LHS = 0 ***)

CONST(CARBO)..  EAT(A) + 6*EAT(B) =G= 60 ; (LHS = 0 ***)

Column Listing      SOLVE MINCOST USING LP FROM LINE 57

Model Statistics    SOLVE MINCOST USING LP FROM LINE 57

MODEL STATISTICS

BLOCKS OF EQUATIONS       2     SINGLE EQUATIONS        4
BLOCKS OF VARIABLES       2     SINGLE VARIABLES        3
NON ZERO ELEMENTS         9

GENERATION TIME      =        0.110 SECONDS

EXECUTION TIME       =        0.170

Solution Report     SOLVE MINCOST USING LP FROM LINE 57

               S O L V E      S U M M A R Y

     MODEL   MINCOST             OBJECTIVE  TOTCOST
     TYPE    LP                  DIRECTION  MINIMIZE
     SOLVER  MINOS5              FROM LINE  57

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE             4400.0000

 RESOURCE USAGE, LIMIT          0.330     5400.000
 ITERATION COUNT, LIMIT         2        10000

 EXIT -- OPTIMAL SOLUTION FOUND


                       LOWER     LEVEL     UPPER    MARGINAL
---- EQU OBJFUNC         .         .         .        1.000

  OBJFUNC     OBJECTIVE FUNCTION


---- EQU CONST       CONSTRAINS

           LOWER     LEVEL     UPPER    MARGINAL

PROTEIN    40.000    40.000     +INF     80.000
CALCIUM    60.000   112.000     +INF       .
CARBO      60.000    60.000     +INF     20.000
```

```
---- VAR EAT         FEED FEEDING LEVELS

     LOWER     LEVEL     UPPER    MARGINAL

A      .       36.000     +INF       .
B      .        4.000     +INF

Solution Report     SOLVE MINCOST USING LP FROM LINE 57

                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR TOTCOST        -INF   4400.000     +INF       .

  TOTCOST     TOTAL COST

**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED

EXECUTION TIME       =        0.110 SECONDS        VERID MW2-25-087


USER: U.S. Department of Agriculture                 G950224:1314CR-MW2
      Washington, DC

**** FILE SUMMARY

INPUT      C:\COURSE\E480\EXP2.GMS
OUTPUT     C:\COURSE\E480\EXP2.LST
```

EXAMPLE 3

The Transportation Problem

One area where linear programming has been especially useful is determining optimal shipping patterns. A typical problem is to determine the minimum cost for shipping output from manufacturing plants to dealers. In general, the problem is to determine how much to ship from each source or supply point to each destination or demand point such that the total shipping costs are minimized. The constraints are the maximum production rates at each source and the quantity demanded at each destination.

Consider the following hypothetical example. An automobile manufacturer with plants in Detroit and Los Angeles must supply its dealers in Atlanta, Chicago, and Denver. Assign an index i to each plant (e.g., 1 for Detroit and 2 for Los Angeles) and an index j to each dealer (e.g., 1 for Atlanta, 2 for Chicago, and 3 for Denver). The data in the following table show the transportation cost per car from each plant to each dealer (i.e., C_{ij}), the maximum production per period at each plant, and the number of cars demanded at each dealership.

	Transportation Cost per Car			
Plant	*Atlanta (1)*	*Chicago (2)*	*Denver (3)*	*Number of Cars Produced (Supply)*
Detroit (1)	200 (C_{11})	100 (C_{12})	300 (C_{13})	3,000
Los Angeles (2)	400 (C_{21})	300 (C_{22})	200 (C_{23})	5,000
Number of cars demanded	3,000	4,000	1,000	

There are six decision variables, X_{ij} (where $i = 1, 2$, and $j = 1, 2, 3$), representing the number of cars shipped from each plant to each dealer. For example, X_{23} would refer to shipments from Los Angeles to Denver. The objective is to determine the number of cars to be shipped from each plant to each dealer that will minimize total shipping costs. That is, minimize

$$C = \sum_i \sum_j C_{ij}X_{ij}$$

The constraints are (1) the total number of cars shipped from each plant (e.g., $X_{11} + X_{12} + X_{13}$ would be total shipments from Detroit) must be equal to or less than the maximum output rate for that plant; and (2) shipments to each dealer (e.g., $X_{11} + X_{21}$ would be the number of cars shipped to Atlanta) must be at least as great as the quantity demanded.

Thus the linear program is to find those values of the decision variables X_{ij} that will minimize

$$C = 200X_{11} + 100X_{12} + 300X_{13} + 400X_{21} + 300X_{22} + 200X_{23}$$

subject to the production or supply constraints

$$X_{11} + X_{12} + X_{13} \leq 3{,}000$$
$$X_{21} + X_{22} + X_{23} \leq 5{,}000$$

the demand constraints

$$X_{11} + X_{21} \geq 3{,}000$$
$$X_{12} + X_{22} \geq 4{,}000$$
$$X_{13} + X_{23} \geq 1{,}000$$

and the nonnegativity constraints

$$X_{ij} \geq 0 \qquad i = 1, 2; \qquad j = 1, 2, 3$$

Clearly, this problem cannot be solved graphically and the algebraic approach would be cumbersome. However, the problem can be solved easily using a microcomputer to yield the following optimal values of the decision variables:

	Least-Cost Shipment Pattern to			
From	*Atlanta*	*Chicago*	*Denver*	*Total Production*
Detroit	$X_{11} = 3{,}000$	$X_{12} = 0$	$X_{13} = 0$	3,000
Los Angeles	$X_{21} = 0$	$X_{22} = 4{,}000$	$X_{23} = 1{,}000$	5,000
Total demand	3,000	4,000	1,000	

Note that the supply (production) and demand constraints are met. For example, total production is 3,000 in Detroit and 5,000 in Los Angeles, which are the maximum output rates in those plants. Further, the dealers in each city receive exactly the number of cars necessary to meet demand. The total transportation cost is $2,000,000. No other set of values for the decision variables would meet the constraints and result in lower transportation costs.

EXAMPLE 3: Model in Mathematical Notation

Minimize $C = \sum_{i=1}^{2} \sum_{j=1}^{3} c_{ij} * Q_{ij}$

Subject to: $\sum_{j=1}^{3} Q_{ij} \leq b_i$

and $\sum_{i=1}^{2} Q_{ij} \geq d_j$ i= 1,2 and j = 1,2,3

Where: c_{ij} is the per unit cost of shipping a car and Q_{ij} is the amount of cars shipped from plant i to dealer j, respectively
b_j is the maximum amount of cars produced at plant i
d_j is the minimum amount of cars needed at dealer j.

EXAMPLE 3: Model in GAMS Notation

```
$TITLE PETERSEN & LEWIS CHAPTER 8, Page 287
* Constrained Cost Minimization: An auto manufacturer with plants in Detroit
* and Los Angeles, wants to minimize its shipping costs of sending cars to dealers in
* Atlanta, Chicago, and Denver, while making sure not to exceed each plant's capacity
* and satisfy the minimum demand for cars from its dealerships.

$OFFSYMLIST OFFSYMXREF
 OPTION NLP = MINOS5

 SETS
   I PLANTS  / DETROIT, LOSANGEL/
   J DEALERS / ATLANTA, CHICAGO, DENVER /

*Data
  PARAMETER
   SUPPLY(I) NUMBER OF CARS PRODUCED
  / DETROIT  3000
    LOSANGEL 5000 /

  DEMAND(J)  NUMBER OF CARS DEMANDED

  / ATLANTA  3000
    CHICAGO  4000
    DENVER   1000 /;

  TABLE
   TRANCOST(I,J) TRANSPORTATION COST PER CAR

              ATLANTA CHICAGO DENVER
  DETROIT     200        100    300
  LOSANGEL    400        300    200 ;

 VARIABLES
   SHIPPED(I,J)  CARS SHIPPED FROM I TO J
   TOTCOST       TOTAL COST OF SHIPPING CARS;

 POSITIVE  VARIABLE SHIPPED;

 EQUATIONS
  OBJFUNC      OBJECTIVE FUNCTION
  SUPPLCON(I)  SUPPLY CONSTRAINT
  DEMANCON(J)  DEMAND CONSTRAINT;

 OBJFUNC ..   TOTCOST =E= SUM((I,J), TRANCOST(I,J)*SHIPPED(I,J));
 SUPPLCON(I) ..  SUM(J, SHIPPED(I,J)) =L= SUPPLY(I);
 DEMANCON(J) ..  SUM(I, SHIPPED(I,J)) =G= DEMAND(J);

 MODEL MINCOST / OBJFUNC, SUPPLCON, DEMANCON /;
 SOLVE MINCOST USING LP MINIMIZING TOTCOST;
```

EXAMPLE 3: **GAMS LIST FILE**

```
GAMS 2.25.087  386/486 DOS                    02/25/98 15:44:47  PAGE 1
PETERSEN & LEWIS CHAPTER 8, Page 287

COMPILATION TIME     =        0.060

Equation Listing    SOLVE MINCOST USING LP FROM LINE 59

---- OBJFUNC     =E=  OBJECTIVE FUNCTION

OBJFUNC..  - 200*SHIPPED(DETROIT,ATLANTA) - 100*SHIPPED(DETROIT,CHICAGO)

- 300*SHIPPED(DETROIT,DENVER) - 400*SHIPPED(LOSANGEL,ATLANTA)

- 300*SHIPPED(LOSANGEL,CHICAGO) - 200*SHIPPED(LOSANGEL,DENVER)+TOTCOST =E= 0; (LHS =
0)

---- SUPPLCON    =L=  SUPPLY CONSTRAINT

SUPPLCON(DETROIT)..  SHIPPED(DETROIT,ATLANTA) + SHIPPED(DETROIT,CHICAGO)

      + SHIPPED(DETROIT,DENVER) =L= 3000 ; (LHS = 0)

SUPPLCON(LOSANGEL)..  SHIPPED(LOSANGEL,ATLANTA) + SHIPPED(LOSANGEL,CHICAGO)

      + SHIPPED(LOSANGEL,DENVER) =L= 5000 ; (LHS = 0)

---- DEMANCON    =G=  DEMAND CONSTRAINT

DEMANCON(ATLANTA)..  SHIPPED(DETROIT,ATLANTA) +     SHIPPED(LOSANGEL,ATLANTA) =G= 3000
; (LHS = 0 ***)

DEMANCON(CHICAGO)..  SHIPPED(DETROIT,CHICAGO) +     SHIPPED(LOSANGEL,CHICAGO) =G= 4000
; (LHS = 0 ***)

DEMANCON(DENVER)..  SHIPPED(DETROIT,DENVER) + SHIPPED(LOSANGEL,DENVER) =G= 1000 ; (LHS
= 0 ***)

Model Statistics    SOLVE MINCOST USING LP FROM LINE 59

MODEL STATISTICS

BLOCKS OF EQUATIONS       3     SINGLE EQUATIONS        6
BLOCKS OF VARIABLES       2     SINGLE VARIABLES        7
NON ZERO ELEMENTS        19

GENERATION TIME      =        0.060 SECONDS

EXECUTION TIME       =        0.110


               S O L V E      S U M M A R Y

     MODEL   MINCOST             OBJECTIVE  TOTCOST
     TYPE    LP                  DIRECTION  MINIMIZE
     SOLVER  MINOS5              FROM LINE  59

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE          2000000.0000

 RESOURCE USAGE, LIMIT          0.220     5400.000
 ITERATION COUNT, LIMIT         4        10000

 EXIT -- OPTIMAL SOLUTION FOUND

                       LOWER     LEVEL     UPPER    MARGINAL
---- EQU OBJFUNC         .         .         .        1.000
```

```
  OBJFUNC     OBJECTIVE FUNCTION

---- EQU SUPPLCON    SUPPLY CONSTRAINT

            LOWER     LEVEL     UPPER    MARGINAL
DETROIT      -INF   3000.000  3000.000  -200.000
LOSANGEL     -INF   5000.000  5000.000      .

---- EQU DEMANCON    DEMAND CONSTRAINT

           LOWER     LEVEL     UPPER    MARGINAL

ATLANTA  3000.000  3000.000     +INF    400.000
CHICAGO  4000.000  4000.000     +INF    300.000
DENVER   1000.000  1000.000     +INF    200.000
Solution Report     SOLVE MINCOST USING LP FROM LINE 59

---- VAR SHIPPED     CARS SHIPPED FROM I TO J

                    LOWER     LEVEL     UPPER    MARGINAL

DETROIT .ATLANTA      .         .        +INF       EPS
DETROIT .CHICAGO      .     3000.000     +INF       .
DETROIT .DENVER       .         .        +INF    300.000
LOSANGEL.ATLANTA      .     3000.000     +INF       .
LOSANGEL.CHICAGO      .     1000.000     +INF       .
LOSANGEL.DENVER       .     1000.000     +INF       .

                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR TOTCOST        -INF  2.0000E+6     +INF       .

TOTCOST     TOTAL COST OF SHIPPING CARS
INPUT      C:\COURSE\E480\EXP3.GMS
OUTPUT     C:\COURSE\E480\EXP3.LST
```

EXAMPLE 4

8-11. The tax commission of a state government employs 150 CPAs, 250 bookkeep and 40 investigators to audit state income tax returns. All employees work 2,0 hours per year. The number of hours required of each type of labor to audit d ferent types of tax returns and the average additional tax revenue collected a result of the audit are as follows:

Type of Return	*Required Time (hours) for:*			*Additional Tax Revenue Collected per Return Audited*
	CPA	*Bookkeeper*	*Investigator*	
Individual	2	4	3	$ 350
Small business	4	7	10	900
Corporation	30	15	24	2,400

a. Set up the linear programming problem to maximize the amount of addition tax revenue collected subject to constraints on time available by each type worker.
b. Solve this problem to determine the revenue-maximizing number of audits each type of return.
c. Set up, solve, and interpret the dual problem.
d. If the agency were faced with a cut in its budget and had to reduce its wo force, which type of worker (i.e., CPA, bookkeeper, or investigator) should the first to be terminated? Explain.
e. What if the state legislature provided additional funds to the tax commissi but specifically directed that they be used to hire five additional CPAs? Ho many additional audits could be performed? Is this decision by the legislatur consistent with economic efficiency? Explain.

EXAMPLE 4: **Model in Mathematical Notation**

Maximize $R = \sum_{i=1}^{3} r_i * X_i$

Subject to: $\sum_{i=1}^{3} a_{ij} * X_i \leq b_j$ i= 1,2,3 and j = 1,2,3

Where: r_i is the additional revenue collected and X_i is the number of returns audited of return type i, respectively
a_{ij} is the time takes to audit return type i by auditor type j
b_j is the total hours of auditor type j available.

EXAMPLE 4: **Model in GAMS Notation**

```
$TITLE PETERSEN & LEWIS CHAPTER 8, similar to PROBLEM 11
* Constrained Tax Return Maximization:  The tax office of a state
* government wishes to determine the number of audits it should conduct
* on Individuals, Small Businesses, and Corporations, given the returns
* resulting from the audits and given the constraints imposed by the
* maximum hours available of its CPA's, Bookkeepers and Investigators.

$OFFSYMLIST OFFSYMXREF
 OPTION NLP = MINOS5
 OPTION RESLIM = 5400

 SETS
   I ENTITIES TO BE AUDITED / INDIV, SMALLBUS, CORP /
   J PERSONNEL  / CPA, BOOKEEP, INV /

*Data
  PARAMETERS
   TAXREV(I) ADDITIONAL TAX REVENUE COLLECTED PER AUDIT
  / INDIV       275
    SMALLBUS    950
    CORP       2200 /

   HOURS(J)  TOTAL HOURS AVAILABLE FOR PERSONNEL
  / CPA      300000
    BOOKEEP  500000
    INV      80000 /

  TABLE
   TIME(I,J) TIME REQUIRED FOR EACH AUDIT BY EACH PERSONNEL

              CPA    BOOKEEP   INV
  INDIV        5        5       0
  SMALLBUS     8       10      10
  CORP        30       15      24 ;

 VARIABLES
   AUDIT(I)  NUMBER OF AUDITS
   TOTTAX    TOTAL ADDITIONAL TAX REVENUE;

 POSITIVE  VARIABLE AUDIT;

 EQUATIONS
  OBJFUNC      OBJECTIVE FUNCTION
  MAXHOURS(J)  MAXIMUM PERSONNEL HOURS;

 OBJFUNC ..   TOTTAX =E= SUM(I, TAXREV(I)*AUDIT(I));
 MAXHOURS(J) ..  SUM(I, TIME(I,J) * AUDIT(I)) =L= HOURS(J);

 MODEL MAXTAX / OBJFUNC, MAXHOURS /;
 SOLVE MAXTAX USING LP MAXIMIZING TOTTAX;
```

EXAMPLE 4: **GAMS LIST FILE**

```
GAMS 2.25.087  386/486 DOS                    02/25/98 15:44:47  PAGE 1
PETERSEN & LEWIS CHAPTER 8, similar to PROBLEM 11

COMPILATION TIME     =        0.050 SECONDS        VERID MW2-25-087

Equation Listing    SOLVE MAXTAX USING LP FROM LINE 59

---- OBJFUNC     =E=  OBJECTIVE FUNCTION

OBJFUNC..  - 275*AUDIT(INDIV) - 950*AUDIT(SMALLBUS) - 2200*AUDIT(CORP) + TOTTAX =E= 0
; (LHS = 0)

---- MAXHOURS    =L=  MAXIMUM PERSONNEL HOURS

MAXHOURS(CPA).. 5*AUDIT(INDIV) + 8*AUDIT(SMALLBUS) + 30*AUDIT(CORP)=L=300000; (LHS= 0)

MAXHOURS(BOOKEEP).. 5*AUDIT(INDIV) + 10*AUDIT(SMALLBUS) + 15*AUDIT(CORP)=L= 500000;
(LHS = 0)

MAXHOURS(INV).. 10*AUDIT(SMALLBUS) + 24*AUDIT(CORP) =L= 80000; (LHS = 0)

MODEL STATISTICS

BLOCKS OF EQUATIONS       2     SINGLE EQUATIONS        4
BLOCKS OF VARIABLES       2     SINGLE VARIABLES        4
NON ZERO ELEMENTS        12

GENERATION TIME      =        0.050 SECONDS

EXECUTION TIME       =        0.110

Solution Report     SOLVE MAXTAX USING LP FROM LINE 59

               S O L V E      S U M M A R Y

     MODEL   MAXTAX              OBJECTIVE  TOTTAX
     TYPE    LP                  DIRECTION  MAXIMIZE
     SOLVER  MINOS5              FROM LINE  59

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE         20580000.0000

 RESOURCE USAGE, LIMIT          0.280     5400.000
 ITERATION COUNT, LIMIT         3        10000

     M I N O S    5.3    (Nov 1990)         Ver: 225-386-02
     = = = = =
     B. A. Murtagh, University of New South Wales
       and P. E. Gill,  W. Murray,  M. A. Saunders and M. H. Wright

EXIT -- OPTIMAL SOLUTION FOUND

                       LOWER     LEVEL     UPPER    MARGINAL

---- EQU OBJFUNC         .         .         .        1.000
  OBJFUNC     OBJECTIVE FUNCTION

---- EQU MAXHOURS    MAXIMUM PERSONNEL HOURS

           LOWER     LEVEL     UPPER    MARGINAL
CPA         -INF  3.0000E+5 3.0000E+5    55.000
BOOKEEP     -INF  3.1600E+5 5.0000E+5      .
INV         -INF  80000.000 80000.000    51.000

---- VAR AUDIT       NUMBER OF AUDITS
```

```
            LOWER     LEVEL     UPPER    MARGINAL
INDIV         .    47200.000     +INF       .
SMALLBUS      .     8000.000     +INF       .
CORP          .         .        +INF   -674.

Solution Report     SOLVE MAXTAX USING LP FROM LINE 59

                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR TOTTAX         -INF  2.0580E+7     +INF       .

  TOTTAX      TOTAL AADITIONAL TAX REVENUE


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED


EXECUTION TIME       =        0.440 SECONDS        VERID MW2-25-087


USER: U.S. Department of Agriculture                 G950224:1314CR-MW2
      Washington, DC

**** FILE SUMMARY

INPUT      C:\COURSE\E480\EXP4.GMS
OUTPUT     C:\COURSE\E480\EXP4.LST
```