

**Please answer all 30 questions. Make sure your answers are as complete as possible. Show all of your work and reasoning. In particular, when there are calculations involved, you must show how you come up with your answers with critical work and/or necessary tables. Answers that come straight from program software packages will not be accepted.**

***You must include the Honor Pledge on the title page of your submitted final exam. Exam submitted without the Honor Pledge will not be accepted.***

***Honor Pledge:*** "I have completed this final examination myself, working independently and not consulting anyone except the instructor. I have neither given nor received help on this final examination."

Use the information below to answer Questions 1 through 3.

Given a sample size of 34, with sample mean 660.3 and sample standard deviation 104.9, we perform the following hypothesis test. Since  $n > 30$ , this is a Z test.

Null Hypothesis                       $H_0 : \mu = 700$

Alternative Hypothesis               $H_a : \mu \neq 700$

1. What is the test statistic? What is the p-value?
2. At a 5% significance level (95% confidence level), what is the critical value(s) in this test? Do we reject the null hypothesis?
3. What are the border values of  $\bar{x}$  between acceptance and rejection of this hypothesis?

**Questions 4 through 7 involve rolling of dice.**

4. Given a fair, six-sided die, what is the probability of rolling the die twice and getting a “1” each time?
5. What is the probability of getting a “1” on the second roll when you get a “1” on the first roll?
6. The House managed to load the die in such a way that the faces “2” and “4” show up twice as frequently as all other faces. Meanwhile, all the other faces still show up with equal frequency. What is the probability of getting a “5” when rolling this loaded die?
7. Write the probability distribution for this loaded die, showing each outcome and its probability.

**Use the data in the table to answer Questions 8 through 9.**

$x$	3	1	4	4	5
$y$	2	-2	5	4	8

8. Determine  $SS_{xx}$ ,  $SS_{xy}$ , and  $SS_{yy}$ .
9. Find the equation of the regression line. What is the predicted value when  $x = 4$ ?

Use the data below to answer Questions 10 through 12.

A group of students from three universities were asked to pick their favorite college sport to attend of their choice: The results, in number of students, are listed as follows:

	Football	Basketball	Soccer
Maryland	60	70	20
Duke	10	75	15
UCLA	35	65	25

Supposed that a student is randomly selected from the group mentioned above.

10. What is the probability that the student is from UCLA or chooses football?
  
11. What is the probability that the student is from Duke, given that the student chooses basketball?
  
12. What is the probability that the student is from Maryland and chooses soccer?

Use the information below to answer Questions 13 and 15.

There are 4000 mangoes in a shipment. It is found that it a mean weight of 15 ounces with a standard deviation of 2 ounces.

13. How many mangoes have weights between 14 ounces and 16 ounces?
  
14. What is the probability that a randomly selected mango weighs less than 14 ounces?
  
15. A quality inspector randomly selected 100 mangoes from the shipment.
  - a. What is the probability that the 100 randomly selected mangoes have a mean weight less than 14 ounces?

- b. Do you come up with the same result in Question 14? Why or why not?
16. Suppose that in a box of 20 iPhone devices, there are 5 with defective antennas. In a draw without replacement, if 3 iPhone devices are picked, what is the probability that all 3 have defective antennas?

Use the information below to answer Questions 17 and 18.

**Benford's law**, also called the **first-digit law**, states that in lists of numbers from many (but not all) real-life sources of data, the leading digit is distributed in a specific, non-uniform way shown in the following table.

<b>Leading Digit</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>Distribution of Leading Digit (%)</b>	<b>30.1</b>	<b>17.6</b>	<b>12.5</b>	<b>9.7</b>	<b>7.9</b>	<b>6.7</b>	<b>5.8</b>	<b>5.1</b>	<b>4.6</b>

The owner of a small business would like to audit its account payable over the past year because of a suspicion of fraudulent activities. He suspects that one of his managers is issuing checks to non-existing vendors in order to pocket the money. There have been 790 checks written out to vendors by this manager. The leading digits of these checks are listed as follow:

Leading Digits	50	15	12	74	426	170	11	23	9
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17. Suppose you are hired as a forensic accountant by the owner of this small business, what statistical test would you employ to determine if there is fraud committed in the issuing of checks? What is the test statistic in this case?
18. What is the critical value for this test at the 5% significance level (95% confidence level)? Do the data provide sufficient evidence to conclude that there is fraud committed?

**Hypothesis Test versus Confidence Interval – Questions 19 through 21**

Random samples of size  $n_1=55$  and  $n_2 = 65$  were drawn from populations 1 and 2 , respectively. The samples yielded  $\hat{p}_1 = .7$  and  $\hat{p}_2 = .6$ .

Test  $H_0: (p_1-p_2) = 0$  against  $H_a: (p_1-p_2) >0$  using  $\alpha = .05$ .

19. Perform a hypothesis test of  $p_1 = p_2$  with a 5% significance level (95% confidence level).
20. Obtain a 95% confidence interval estimate of  $p_1 - p_2$ .
21. Do you come up with the same conclusion for Question 19 and Question 20? Why or why not?

**Hardness of Gem – Questions 22 and 23**

Listed below are measured hardness indices from three different collections of gemstones.

Collection	Hardness Indices													$\bar{x}_i$	$s_i^2$
A	9.3	9.3	9.3	8.6	8.7	9.3	9.3	--	---	---	---	---	---	9.91	0.10
B	8.7	7.7	7.7	8.7	8.2	9.0	7.4	7.0	---	---	---	---	---	8.03	0.60
C	7.2	7.9	6.8	7.4	6.5	6.6	6.7	6.5	6.5	7.1	6.7	5.5	7.3	6.82	0.34

You are also given that  $\bar{x} = 7.99$ .

22. What is the test statistic?
23. Use a 5% significance level (95% confidence level) to test the claim that the different collections have the same mean hardness.

24. The probability that an individual egg in a carton of eggs is cracked is 0.03. You have picked out a carton of 1 dozen eggs (that's 12 eggs) at the grocery store. Determine the probability that at most one of the eggs in the carton are cracked.
25. In a group lineup of 7 models in a commercial, 3 are male and 4 are female. In how many ways can you arrange 3 models in a lineup if the first and the third must be a male but the second one must be a female?
- 26.

Pair	Sample from Population 1 (observation 1)	Sample from Population 2 (observation 2)
1	7	4
2	3	1
3	9	7
4	6	2
5	4	4
6	8	7

The data for a random sample of six paired observations are shown in the table above.

- Compute  $\bar{d}$  and  $S_d$
- Express  $\mu_d$  in terms of  $\mu_1$  and  $\mu_2$ .
- Form a 95% confidence interval for  $\mu_d$ .
- Test  $H_0: \mu_d = 0$  against  $H_a: \mu_d \neq 0$ . Use  $\alpha = .05$

27. Peter, Paul, Mary, John and Martha are members of the pastoral council at a local church. They are to be seated at one side of a long conference table in a pastoral council meeting.

- a) How many possible ways can these 5 council members be seated?
- b) How many possible sitting arrangements are there if only gender is considered in the process?

28. How many social robots would need to be sampled in order to estimate the proportion of robots designed with legs, no wheels to within .075 of its true value with 99% confidence. Given that a random sample of 106 robots showed that 63 were designed with legs, no wheels.

29. Composite sampling is a way to reduce laboratory testing costs. A public health department is testing for possible fecal contamination in public swimming pools. In this case, water samples from 5 pools are combined for one test, and further testing is performed only if the combined sample shows fecal contamination. Based on past experience, there is a 3% chance of finding fecal contamination in a public swimming area. What is the probability that a combined sample from 5 swimming pools has fecal contamination? Recall that  $P(A) + P(\text{not } A) = 1.0$ .

30 A random sample of five accidents resulted in the following number of persons injured: 18, 15, 12, 19, & 21. Using the .01 significance level, can we conclude the population mean is less than 20 for all accidents?

- a) State  $H_0$  and  $H_a$ ?
- b) Test statistic = ?
- c) Critical value = ?
- d) Reject  $H_0$ : (yes or no)