A Course in Public Economics: Resources for Instructors

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Course Design

Overview

A review of the contents and methodology of each chapter follows. The number in the middle column is an estimate of the chapters difficulty, on a 1–5 scale with 1 being the easiest. For this purpose, I have imagined the students to be in their third or fourth year, with one or two terms of Varian-style intermediate microeconomics, and with a grasp of basic calculus but little exposure to statistics.

1: Introduction	1	Discusses the two fundamental theorems of welfare economics. Informally states the requirements of the first theorem, and describes the market failures that arise when these requirements are not satisfied: public goods, externalities, imperfect competition. Introduces the concept of asymmetric information, and discusses its relevance to the first theorem. Argues that asymmetric information precludes the government from transferring income in a lump sum fashion, so that the requirements of the second theorem are also unlikely to be satisfied. This chapter are entirely verbal.
2: The Exchange Economy	2	Introduces the concept of Pareto optimality and illustrates it with a simple example. Develops the Edgeworth box, and uses it to discuss Pareto optimality and competitive equilibrium within the context of an exchange economy with two people and two goods. Demonstrates the two theorems for this economy. The arguments are verbal and graphical.
3: An Algebraic Exchange Economy	2	Constructs an algebraic version of the two person, two good exchange economy. Describes Pareto optimal allocations as solutions to an under-determined equation system and presents a Cobb-Douglas example. Describes competitive equilibrium as the solution to an equation system and presents a Cobb-Douglas example. Demonstrates the two theorems by comparing the two equation systems.

4:	The Production Economy	3	Develops a two person, two good economy in which the people are endowed with two factors of production and the ownership of firms, but not with consumer goods. Describes a Pareto optimal allocation as one satisfying three efficiency conditions, and discusses each efficiency condition. Shows that a Pareto optimal allocation can be represented as a solution to an under-determined equation system. Describes competitive equilibrium, and shows that it can also be thought of as the solution to an equation system. Presents an example of competitive equilibrium involving simple functional forms. Demonstrates the first theorem by comparing the equation systems.
5:	Consumer and Producer Surplus	2	Shows that the increase in total cost when output rises can be measured by an area under the marginal cost curve. Presents a more general rule of the relationship between margins and totals. Introduces consumer and producer surplus, and uses the general rule to measure them. Argues that a system of free markets maximizes surplus, and that interventions cause a loss of surplus, this being the intervention's welfare cost. Examines the welfare cost of a quantity constraint, and argues that the welfare cost measures the value of the mutually beneficial trades that are prevented by the intervention. Introduces taxes and subsidies, and shows that these interventions involve a transfer of surplus between the government and the private sector. Shows that both interventions reduce surplus: the welfare cost of the tax arises because mutually beneficial trades are discouraged; and the welfare cost of the subsidy arises because trades that are not mutually beneficial are encouraged. Discusses the measurement of the welfare cost of a tax when there is more than one tax. Primarily verbal and graphical, but there is some algebra.
6:	Externalities and Negotiation	1	Discusses the problem of externalities. Presents the results commonly known as the Coase Theorem: if the parties affected by an externality successfully negotiate, they reach an efficient outcome under either assignment of the property rights; the assignment of the property rights affects the distribution of income but has no efficiency implications. The reasons why negotiations might fail are then explained, and the government policies that could correct an externality are discussed. The Pigouvian tax is introduced. The arguments are predominantly verbal and graphical.

7: Permit Trading	3	Presents a model in which two firms have different abatement costs. Calculates the resource cost of abatement under proportional emissions reductions. Introduces the idea of tradable permits, and derives the equilibrium price of permits. Shows that the resource cost of abatement is smaller under permit trading than under proportional reductions. The arguments are predominantly algebraic.
8: Renewable Common Property Resources	3	Sets up a static model of the fishery under competition and management. Shows that the competitive outcome is inefficient, and identifies the negative externality that causes this result. Shows that the externality can be corrected by a Pigouvian tax. A dynamic model of the fishery is then developed, and the depletion of the fishery under competition is demonstrated. Alternative management regimes are described. The possibility of extinction is briefly discussed. The models are both algebraic and graphical. The dynamic model introduces the idea of a difference equation (but not its solution).
9: Co-ordination Failures	3	Identifies situations in which each agent's actions have positive external effects on the welfare of the other agents. Introduces the idea of a strategic game, and sets up a co-ordination game in which each of two firms chooses between two actions (produce and don't produce). Shows that the game can be represented by a two-by-two table. Introduces the idea of a Nash equilibrium, and demonstrates that the game has two Nash equilibria, and that both firms prefer one Nash equilibrium to the other. Develops a variant of the game involving uncertainty: each firm's production cost is an independent draw from a uniform distribution. Shows that there is a Nash equilibrium in which each firm produces only when its costs are sufficiently low. Shows that the Nash equilibrium is unique and inefficient. The game with certainty is simple. The game with uncertainty involves the calculation of a number of expected values.
10: Pure Public Goods	2	Introduces the concept of a pure public good. Graphically derives the Pareto optimal allocation for a two person economy with one private good and one public good, and discusses the Samuelson condition. Develops the voluntary contributions model for public goods provision, and finds the Nash equilibrium for the two person game. Invokes symmetry to find the Nash equilibrium in the n person game. Compares the Pareto optimal provision of a public good with its provision under voluntary contributions, and discusses free riding. The latter part of the chapter is predominantly algebraic.

11: Two Examples of Pure Public Goods	3	The first section identifies knowledge as a public good and argues that too little of society's resources will be allocated to its provision. Corrective government policies are discussed, with an emphasis on the effects of patents. The arguments are entirely verbal. The second section presents a model in which the wealthy care about their own incomes and the incomes of the poor. It is shown that some income distributions are Pareto optimal and that some are not. A game in which wealthy people choose their charitable contributions is developed, and it is shown that the contributions of the wealthy are never large enough to shift the economy to a Pareto optimal income distribution: each wealthy person (and each poor person) would be better off if every wealthy person contributed a little more. It is then shown that charity has the properties of a pure public good, and that the insufficiency of the charitable contributions is another manifestation of the free rider problem. This game is more complicated and involves more extended reasoning that the games presented in the earlier chapters.
12: Impure Public Goods	4	Discusses two simple forms of impure public good. A club good is assumed to be congestible, replicable and excludable. The optimal size and membership of a club good are characterized. A variable-use public good is assumed to be congestible and non-replicable. The particular case of a highway is discussed. The optimal capacity of the highway in the absence of tolls is calculated. The optimal capacity in the presence of tolls is also calculated. The optimal toll is shown to be the Pigouvian tax. The arguments are predominantly algebraic.
13: The Link Between Public Goods and Externalities	4	Argues that an essential feature of both public goods and externalities is the interdependence of preferences. Develops the Shibata box diagram, and shows that the Samuelson condition describes an optimal allocation in both cases. Discusses bargaining and voluntary contributions as alternative ways of determining the outcome. Although predominantly diagrammatic, this chapter is conceptually difficult.

14: Monopoly	2	Discusses natural monopoly and demonstrates the welfare cost of monopoly. Shows that the welfare cost can be reduced through regulation. Shows that the welfare cost can be eliminated through government ownership, but argues that this gain might be offset by the need to raise more revenue through distortionary taxes. Introduces the concept of rent-seeking, and develops a game in which n firms expend scarce resources in an effort to become the monopolist. The Nash equilibrium has the property that the total resources expended approach the present value of monopoly profits as n grows arbitrarily large.
15: Pricing Rules under Imperfect Competition	3	Discusses the role of marginal cost pricing in the first theorem. Verbally discusses Bertrand equilibrium, collusion, and the Kreps and Scheinkman's capacity constraints game. Presents in detail a simplified version of Dixit and Stiglitz's model of monopolistic competition. The emphasis in both cases is on whether imperfectly competitive firms adopt marginal cost pricing.
16: Taxation	3	Reviews the concept of distortionary taxation, and asks whether there is a system of commodity taxes that is equivalent to a lump sum tax. Develops a two good, n person production economy in which the government raises a fixed amount of revenue. Shows that, if the supply of labour is fixed, the welfare cost of taxation is zero if both commodities are taxed at equal rates. Shows that, if the supply of labour is variable, the welfare cost of taxation is zero only if both commodities are taxed at equal rates *and* market work is subsidized at the same rate. However, this configuration of taxes raises no revenue, leading to the conclusion that any tax system that raises positive revenue generates a positive welfare cost. Concludes by characterizing the optimal size of government in the presence of distortionary taxation. Except for the last section, the arguments are algebraic and relatively abstract.
17: The Welfare Costs of Tax Interactions	4	Examines a two good, one person production economy in which taxes are imposed first on one good and then on both. The first part of the chapter examines the competitive equilibria associated with these tax structures, and calculates the individual's utility under each structure. The second part of the chapter calculates the general equilibrium welfare costs associated with each tax structure, and shows that the welfare costs mirror the utility changes. The first part of the chapter is algebraic. The second part is verbal and graphical, but conceptually challenging, and would be the more difficult of the two for many students.

18: The Theory of the Second Best	4	Develops a two good, one person production economy to study the optimal tax structure. Argues that the optimal tax structure has the property that the last dollar of revenue raised through each tax reduces the person's utility by the same amount. Shows that this rule implies the inverse elasticity rule, the "equal proportionate reductions in output" rule, and a kind of Corlett and Hague rule. The last half of the chapter derives the Ramsey pricing rule, which characterizes the optimal pricing of the output of a natural monopoly, under either regulation or government ownership, when taxation is distortionary. The treatment of both topics is predominantly algebraic.
19: Asymmetric Information	2	Introduces some of the basic concepts of asymmetric information. Develops Akerlof's lemons model as an example of adverse selection. Develops a static game of theft and deterrence as an example of moral hazard. Both models are relatively simple, although the lemons model makes use of the uniform probability distribution.
20: Preference Revelation	5	Develops a model in which individuals differ in their preferences over public and private goods. Introduces the idea of a Lindahl equilibrium. Shows that the Lindahl equilibrium cannot be implemented if the government cannot observe preferences. Shows that an equilibrium in which everyone is assigned the same tax price also leads to preference revelation problems. Shows that the Groves-Clarke mechanism elicits truthful revelation. Discusses the problem of excess revenue.
21: Regulation of a Natural Monopoly	5	Presents Laffont and Tirole's model of natural monopoly, in which costs and managerial effort cannot be observed by the regulator. Explains the regulator's inability to implement the full information equilibrium, and describes the best asymmetric information equilibrium. Some of the more technical material is relegated to an optional appendix.
22: Other Examples of Asymmetric Information	2	Discusses the implications of asymmetric information for health care and health care insurance. Discusses the implications of asymmetric information for the loan market, shows that the "standard debt contract" is the best attainable contract, and describes the consequences of asymmetric information for the allocation of the economy' resources. Presents Shapiro and Stiglitz's model of efficiency wages as an example of the principal-agent problem. The arguments are often verbal, and the more technical aspects of these issues are avoided.

23: The Distribution of Income	1	A general discussion of basic aspects of income distribution, namely the determinants of the income distribution, the relationship between welfare and income, and the rationale for income redistribution. The arguments are predominantly verbal.
24: The Limits to Income Redistribution	4	Develops a model in which workers choose between work in a high-wage industry and a low-wage industry based upon their personal attributes. Shows that the government can equalize incomes without reducing the efficiency of the economy if these attributes are observable. Shows that, if these attributes are not observable, attempts to redistribute income cause some workers to move to the low-wage industry (where they are less productive), impairing economic efficiency and ultimately limiting the degree of income redistribution that can occur. This model makes use of the uniform probability distribution.
25: Tagging and Targeting	5	Examines two kinds of policies that allows the government to minimize the efficiency loss that occurs when it attempts to redistribute income under conditions of asymmetric information. The discussion of tagging is verbal, but the models of targeting are algebraic and involve questions of incentive compatibility.
26: The Role of Government in a Market Economy	1	Briefly reviews the government's potential role in the correction of market failure and the redistribution of income. Concludes with a discussion of the factors that limit the government's ability to carry out this role.

Course Outline

This textbook is intended for students who have had at least one term of intermediate microeconomics, some exposure to multivariate calculus and minimal exposure to statistics. For students in this target group, a thirteen week course that includes the following is possible but demanding:

1: Introduction
2: The Exchange Economy
3: An Algebraic Exchange Economy
5.1–5.3: Consumer and Producer Surplus
6: Externalities and Negotiation
8.1: The Static Common Property Problem
9: Co-ordination Failures
10: Pure Public Goods
11: Two Examples of Pure Public Goods
14: Monopoly

Some parts of the book will be routine for students with stronger backgrounds, and a more

demanding course should be designed for them. Students with weaker backgrounds are likely to flounder in any course based upon this textbook.

Students who have had minimal exposure to problem-oriented microeconomics are initially unable to solve the problems at the end of the chapters on their own. I accommodate them in two ways:

- I devote a number of lectures to solving the problems at the end of the chapters. I expect the students to decide what steps need to be taking, giving them hints when they get off track or run out of ideas. These sessions are not very useful if the students do not actively participate, and large classes are likely to encounter a "free rider problem" in which a few students participate and the rest simply copy down the answers. I deal with this problem by limiting each student to one contribution on each question, and when the contributions stop coming, I leave the remainder of the problem for the students to do on their own. They quickly decide that they'd rather do the problems with my help than without it, and behave accordingly.
- I give two and sometimes three midterm exams. The students prefer multiple midterms because it gives them practice at writing exams, and because it reduces the impact of a bad exam.

The textbook is written so that many parts of the book stand alone, so that course outlines that emphasize other subjects can easily be constructed. The remainder of this section outlines the options that are available in the various parts of this course.

Introduction The introduction provides an overview of the textbook, and provides a framework within which the individual parts of the course can be fitted. Students should be strongly encouraged to read this chapter.

Markets Students should be familiar with chapters 2 and 3 and with sections 5.1–5.3 before attempting the rest of the book. The amount of time that must be spent on this material will depend upon the background of the students. Students with even one term of intermediate microeconomics will have been exposed to the material in sections 5.1–5.3 (but are likely to be in need of review), while students who have had two terms of microeconomics will already have encountered much of the material contained in chapter 2. Chapter 3 is pivotal, because it presents the methods used in the remainder of the course. I spend a considerable amount of time on this chapter, and work through most of the problems. I find that this part of the course takes about four weeks.

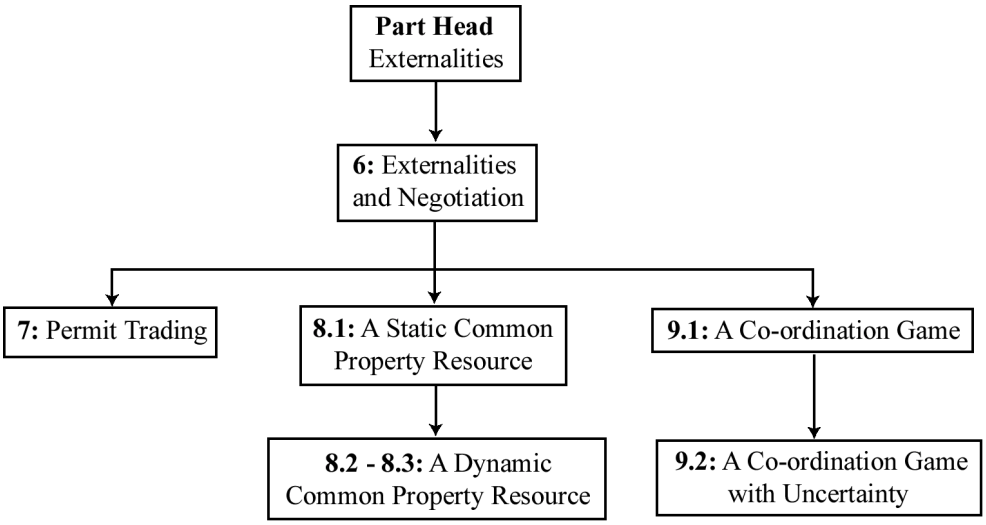
Chapter 4 extends the concepts of Pareto optimality and competitive equilibrium to the production economy, and proves the first theorem for this kind of economy. It can be included in courses that emphasize the positive attributes of the market system; but if the emphasis is on market failure, it can be omitted or presented in a truncated form. In the latter case, it is useful to point out to the students that a Pareto optimal allocation in the two-person production economy satisfies the condition:

$$MRS_1 = MRS_2 = MRT$$

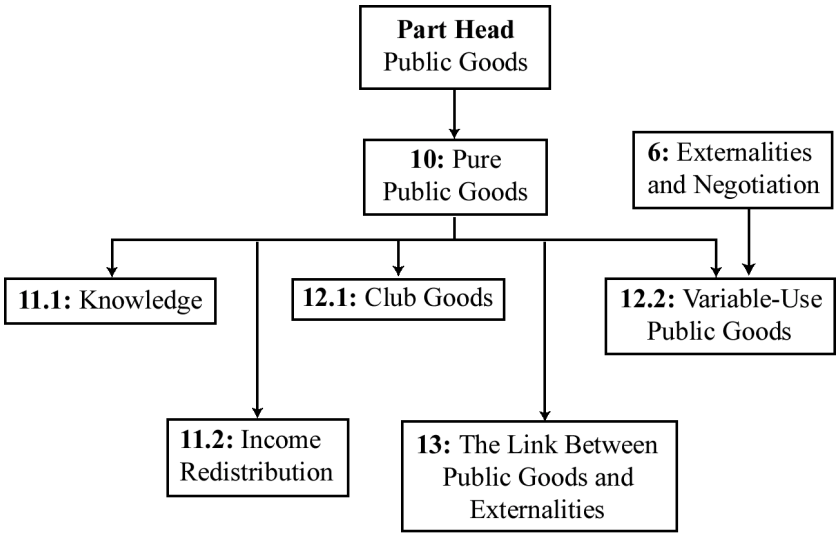
This condition can then be compared to the Samuelson condition when public goods are introduced. The fastest way to obtain this result is to simply assume that the economy produces

a combination of goods lying on the production possibility frontier. This assumption means that the issue of production efficiency need not be addressed. The students are already familiar with the first half of this condition, so only the second half needs to be explained—see the discussion of "match efficiency" in section 4.1. This assumption is employed in some of the end-of-chapter problems.

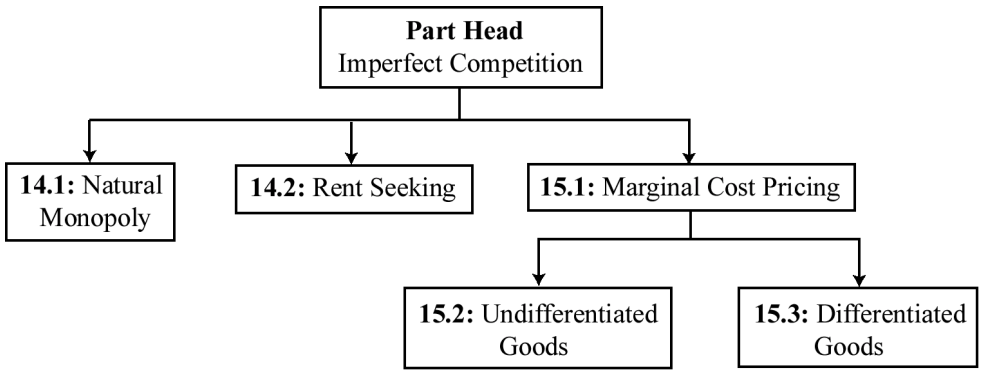
Externalities The part head and chapter 6 contain all of the basic material on externalities. The remaining material does not contain any "building blocks" that become important in later chapters, so they can be skipped without adverse consequences. The flow chart below lays out the options in this section. Topics lower down in the diagram require topics higher in the diagram, so you can trace the arrows backwards to find the prerequisites for any topic.



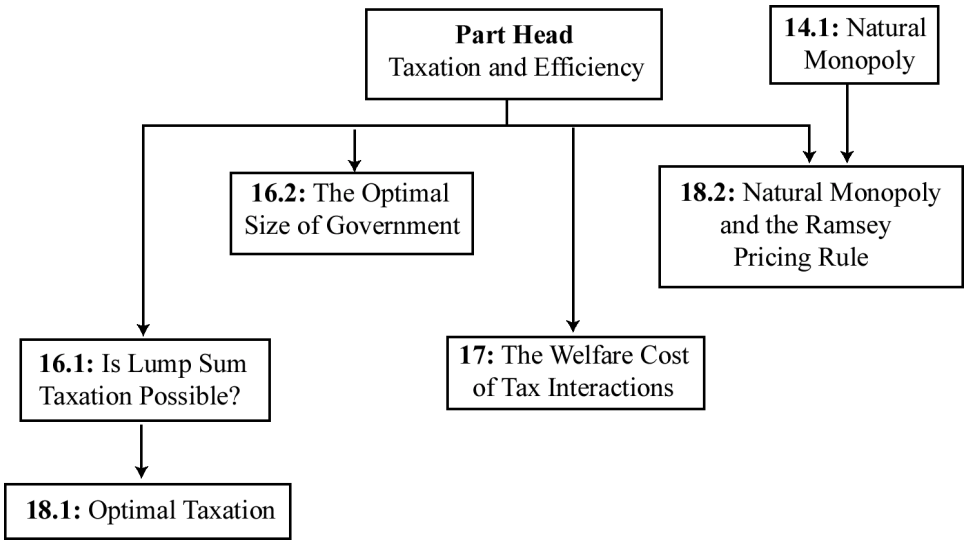
Public Goods The part head and chapter 10 contain the basic material. Any or all of the remaining four topics can be included or excluded without adverse consequences. The flow chart on the next page lays out the options. (Section 12.2 makes use of the Pigouvian tax and some other concepts set out in chapter 6.)



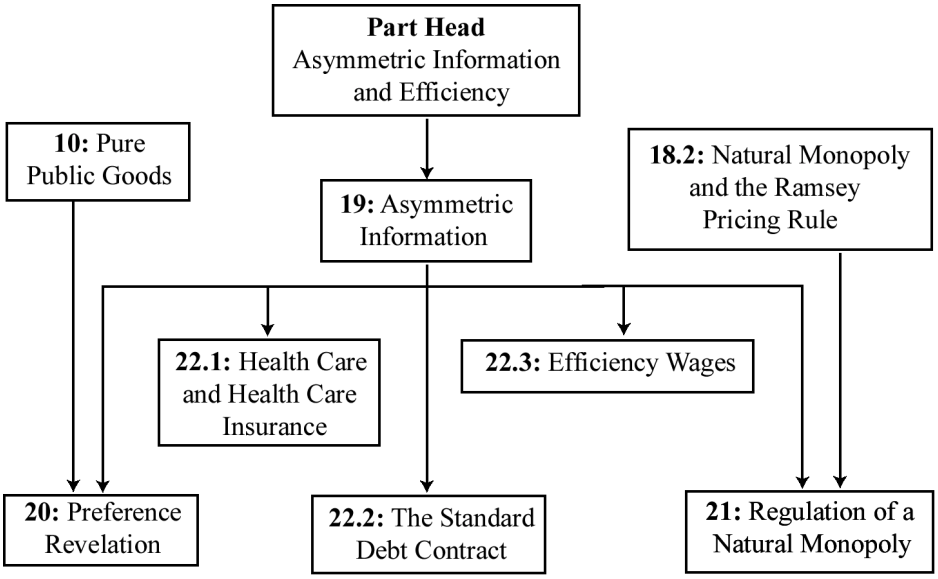
Imperfect Competition This part consists of a number of independent topics. Natural monopoly is a prerequisite of the Ramsey pricing rule (in chapter 18) and regulation under asymmetric information (in chapter 21).



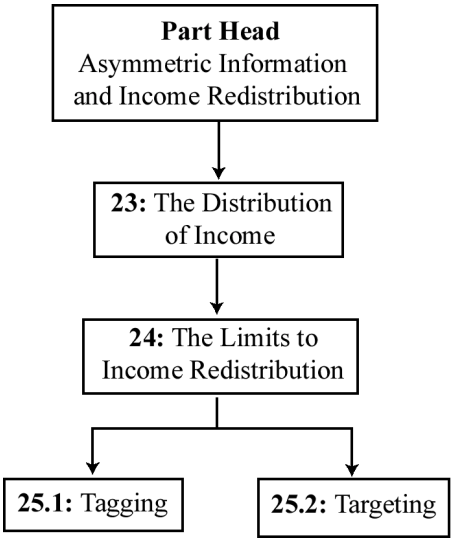
Taxation and Efficiency Again, this part has a more or less lateral organization. Section 18.2 is a prerequisite of chapter 21. Section 16.1 is a useful precursor of chapter 24, but is not necessary for it.



Asymmetric Information and Efficiency The part head and chapter 19 lay out the basic concepts, and the remaining topics are all optional. Note, however, that chapters 20 and 21 have additional prerequisites.



Asymmetric Information and Income Redistribution Unlike the previous parts of the book, this part will make most sense if developed in the order laid out in the textbook.

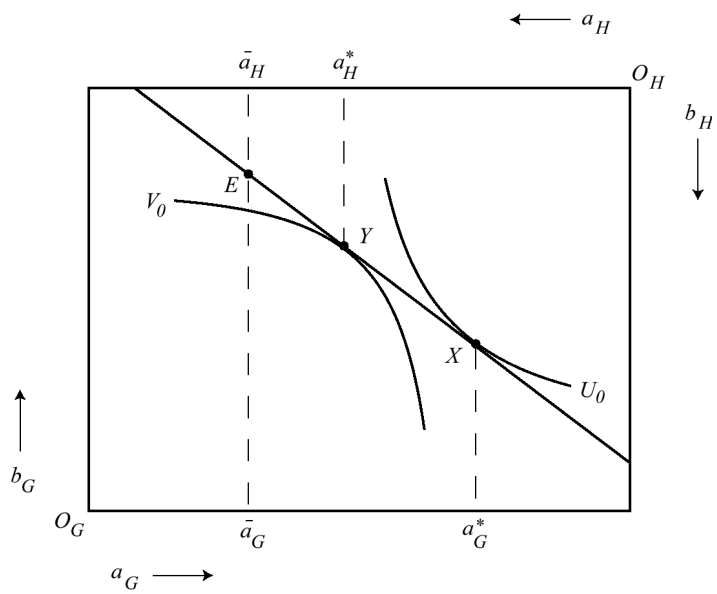


Solutions to Textbook Questions

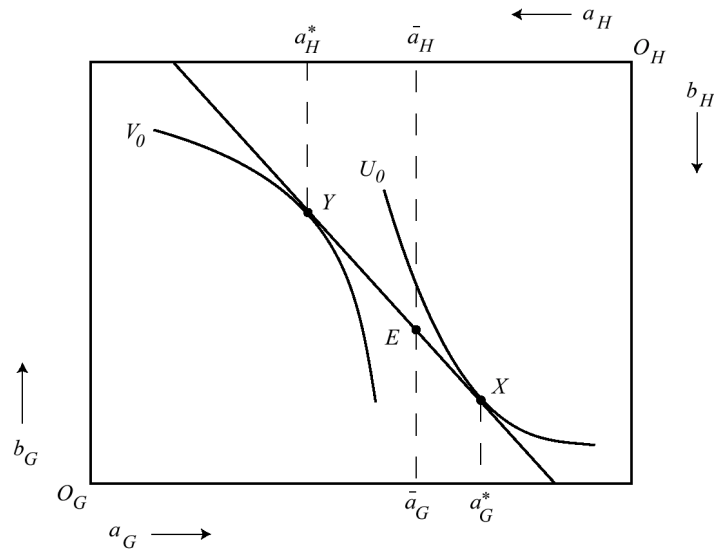
Chapter 2: The Exchange Economy

Question 1 In parts a) and b), E is the endowment point, X is George's best attainable commodity bundle, and Y is Harriet's best attainable commodity bundle. In part c), E is the endowment point, and at the current price, neither George nor Harriet wishes to trade away from this point.

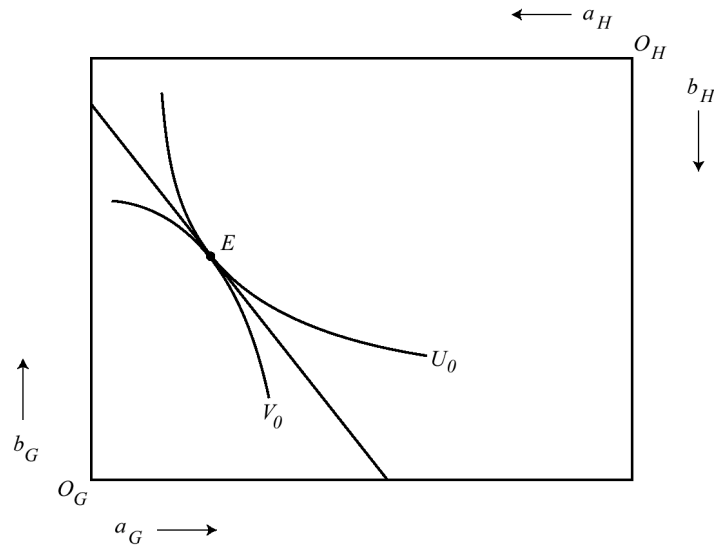
a)



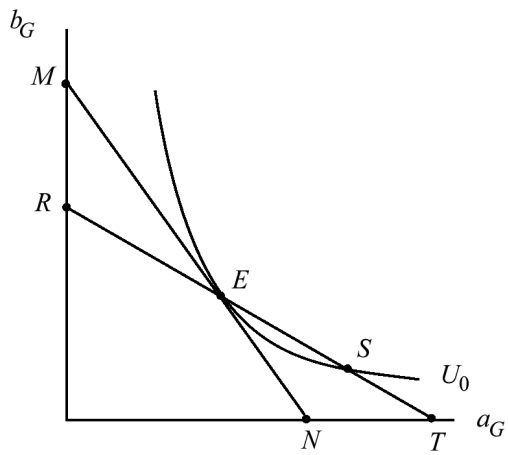
b)


a_H
a_H^*
$\bar{a}_H$
O_H
b_H
V_0
U_0
Y
E
X
b_G
O_G
$\bar{a}_G$
a_G^*
a_G

c)


a_H
O_H
b_H
E
U_0
V_0
b_G
O_G
a_G

Question 2 In the figure below, George is endowed with the commodity bundle E. The budget line MN shows the commodity bundles that George could obtain by trading at the price p^*. As required, George maximizes his utility (reaching the indifference curve U_0) when he chooses not to trade. If the price were less than p^*, the budge line would be flatter but would again pass through the endowment point E. The budget line RT corresponds to one of these lower prices. If faced with this budget line, George could acquire any of the commodity bundles lying on the segment RE by selling ale for bread. He prefers the bundle E to any of these alternatives, however, because E lies on the indifference curve U_0 while every bundle on RE lies on a lower indifference curve. Alternatively, George could acquire one of the bundles on the segment ET by selling bread for ale. Every commodity bundle lying on the budget line between E and S is better, in George's opinion, than the bundle E, and hence George will choose to acquire one of these bundles. That is, at any price less than p^*, George will choose to sell bread for ale. A similar argument shows that, at every price greater than p^*, George will prefer to sell ale for bread.



Question 3 a) Both allocations can be Pareto optimal. Suppose that the allocations are identified by the numbers 1 and 2, and that Monday's and Tuesday's preferences are described by the Table below. Allocation 1 is Pareto optimal because Monday would be worse off under the only alternative allocation; and allocation 2 is Pareto optimal because Tuesday would be worse off under the only alternative allocation.

Monday	**Tuesday**
1	2
2	1

b) All of the allocations can be Pareto optimal. Let Monday's and Tuesday's preferences over allocations 1–5 be given by the table below:

Monday	Tuesday
1	5
2	4
3	3
4	2
5	1

Consider allocation 2. The alternatives to this allocation are 1, 3, 4 and 5. A switch from 2 to 1 would make Monday better off but Tuesday worse off. A switch from 2 to any of the other alternatives would make Tuesday better off but Monday worse off. Since there is no alternative to 2 at which someone would be better off and no-one would be worse off, 2 is Pareto optimal. The same argument can be applied to each of the other allocations.

c) There must be a Pareto optimal allocation. The following result can be used to demonstrate this fact.

RESULT: Imagine that there are n possible allocations, identified by the integers 1 through n; and imagine that allocation m is Pareto optimal. If the set of possible allocations is expanded to include a new allocation (call it allocation $n+1$), at least one of the allocations m and $n+1$ is Pareto optimal.

PROOF: There are two possibilities that have to be considered.

Possibility A. A switch from m to $n+1$ makes someone better off without making anyone else worse off.

Possibility B. A switch from m to $n+1$ *does not* make someone better off without making anyone else worse off

Consider B first. Since m was initially Pareto optimal, a switch from m to any of the other original allocations does not make someone better off without making anyone else worse off. Under B, a switch from m to $n+1$ also does not make someone better off without making anyone worse off. Thus, there is still no alternative to m at which someone is better off and no-one is worse off—that is, m is still Pareto optimal—when B holds.

Now consider A. Since $n+1$ is an alternative to m at which someone is better off and no-one is worse off, m is no longer Pareto optimal. However, $n+1$ is itself Pareto optimal if none of the original n allocations is "better" than $n+1$ in the sense that someone is better off and no-one is worse off. Under A, m is not better than $n+1$ in this sense. Also, none of the other original allocations is better than $n+1$ in this sense:

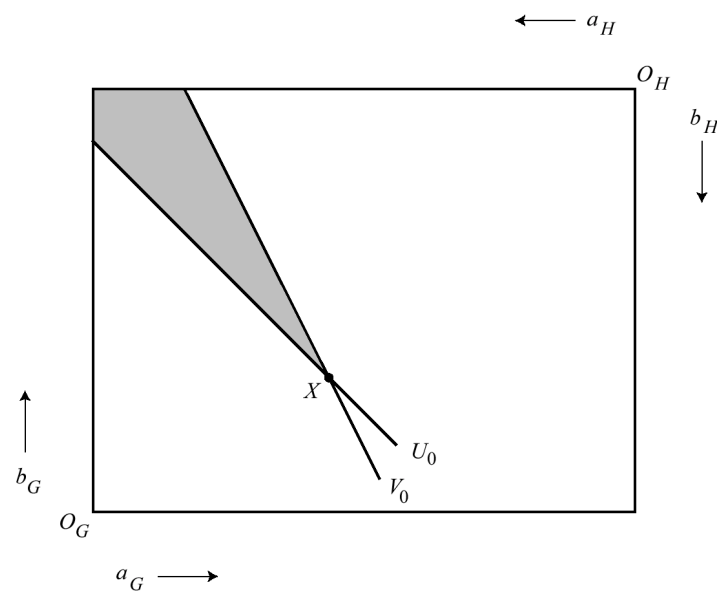
> Since m was originally Pareto optimal, none of the other original allocations is better than m in the sense that someone is better off and no-one is worse off. Since each person believes that $n+1$ is at least as good as m, none of these allocations is better than $n+1$ in the same sense.

Thus, $n+1$ satisfies the requirements of Pareto optimality when A holds. ■

This result can be applied recursively to show that there must be a Pareto optimal allocation. If allocation 1 were the only possible allocation, it would be Pareto optimal (there would be no "better" alternative because there would be no alternatives). Now imagine that the set of possible allocations is extended by adding allocation 2. The result implies that at least one of these two allocations is Pareto optimal. Now expand the set of possibilities by adding

allocation 3. The result now implies that at least one of these three allocations is Pareto optimal. Proceeding in this fashion until all 108 allocations are included in the set of possible allocations shows that at least one of these allocations is Pareto optimal.

Question 4 a) The required region lies above and to the left of X, bounded by the indifference curves passing through X, and by left and top sides of the box. That is, it consists of the shaded area and its boundary in the diagram below. The allocations within this region have these properties: *i*) if the allocation lies on George's indifference curve U_0, George believes that it is just as good as X and Harriet believes that it is better than X; *ii*) if the allocation lies on Harriet's indifference curve V_0, Harriet believes that it is just as good as X and George believes that it is better than X; *iii*) if the allocation lies in the interior of the box or on an edge of the box, both people believe it to be better than X.



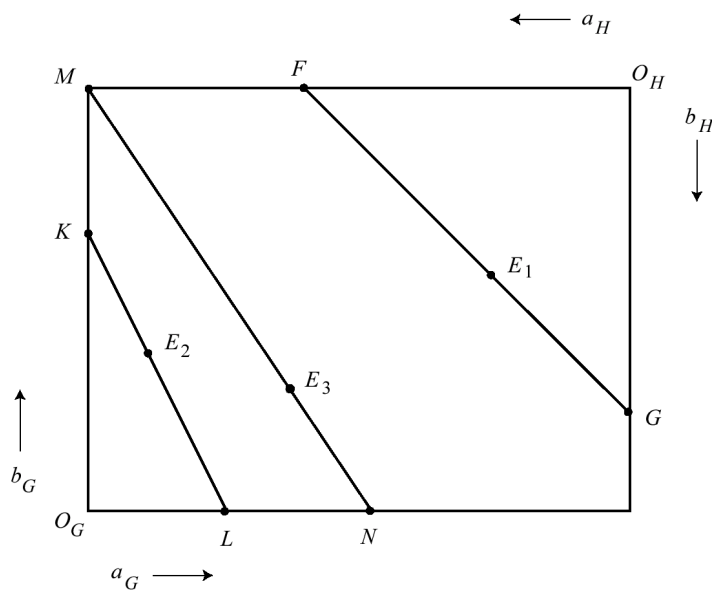
If U_0 had cut the top of the box, or if V_0 had cut the side of the box, this region would have been triangular rather than diamond-shaped.

b) If the indifference curves passing through an allocation delimit a region like the one shown above, that allocation is not Pareto optimal. Since a region of this kind is associated with every allocation inside the box, or on its right or bottom edges, none of these allocations is Pareto optimal. No such region exists for allocations on the top or left edge of the box, so for these allocations, there are no alternatives at which someone is better off and no-one is worse off. They are therefore Pareto optimal.

c) If p is equal to one, the budget line coincides with one of George's indifference curve. George believes that every commodity bundle on the budget line is equally good, and is therefore willing to engage in any trade at that price. If p is greater than 1, George wants to sell all of his ale for bread. Similarly, if p is equal to two, the budget line coincides with one of Harriet's indifference curves. Harriet believes that every commodity bundle on the budget

line is equally good, and is therefore willing to engage in any trade at that price. If p is less than two, Harriet wants to sell all of her bread to obtain ale.

d) These possibilities are illustrated in the diagram below, in which FG and KL are budget lines with slopes -1 and -2 respectively. The slope of the final budget line, MN, is between -1 and -2.



If the endowment point is E_1, the budget line with slope -1 cuts the top of the box, as required by i). The results of c) tell us how George and Harriet would behave if confronted by this budget line. Harriet would want to sell bread for ale until she had no more bread—that is, until the allocation F has been reached. George is indifferent between all of the commodity bundles lying on this budget line, so he is content with a trade that takes him to F. Thus, the price 1 is market-clearing.

If the endowment point is E_2, the budget line with slope -2 cuts the left side of the box, as required by ii). Confronted by this budget line, George would want to sell ale for bread until he had no more ale. Harriet, on the other hand, is indifferent between the commodity bundles lying on this budget line, and so is content to take the other side of the trade. This price is market-clearing, and under this price, trade takes the economy to the allocation K.

Finally, suppose that the endowment point is E_3, and consider the budget line that runs to the top left corner of the box. By construction, its slope is between -1 and -2. George wants to sell ale for bread until he runs out of ale, and Harriet wants to sell bread for ale until she runs out of bread. Since George runs out of ale exactly when Harriet runs out of bread, both are happy with a trade that takes them to the allocation M. Thus, the current price is a market-clearing price.

Chapter 3: An Algebraic Exchange Economy

Question 1 a) Harold's endowment of bread is 8 loaves.

b) If Harold wishes to consume a pints of ale, he is able to sell $6 - a$ pints of ale. The bread value of this ale is $p(6 - a)$ loaves.

c) If Harold wishes to consume c pounds of cheese, he is able to sell $10 - c$ pounds of cheese. The bread value of this cheese is $q(10 - c)$ loaves.

d) Harold's actual consumption is the sum of these three amounts, so his budget constraint is:

$$b = 8 + p(6 - a) + q(10 - c)$$

Question 2 a) The equilibrium price is the price at which the sum of the excess demands is equal to zero:

$$ED_{At} + ED_P + ED_{Ar} = 0$$

Substituting the expressions for the excess demands into this equation and simplifying gives:

$$\frac{24}{p} - 8 = 0$$

$$p^* = 3$$

Evaluating the excess demands at the equilibrium price gives:

$$ED_{At} = -4/3$$

$$ED_P = -2/3$$

$$ED_{Ar} = 2$$

Since a positive excess demand indicates a purchase and a negative excess demand indicates a sale, Athos sells 4/3 of a chicken, Porthos sells 2/3 of a chicken, and Aramis buys 2 chickens.

b) Chickens are being traded for bread. Anyone who is selling chickens is simultaneously buying bread, and anyone who is buying chickens is selling bread. Since the equilibrium price of a chicken is 3 loaves of bread, Athos buys 4 loaves of bread, Porthos buys 2 loaves, and Aramis sells 6 loaves.

Question 3 a) Dinah is endowed with 10 loaves of bread, but she must spend pa_D loaves of bread to purchase a_D pints of ale. Consequently, the relationship between her consumption of bread and her consumption of ale (*i.e.,* her budget constraint) is:

$$b_D = 10 - pa_D \tag{1}$$

Trade at the price p allows her to purchase any commodity bundle (a_D, b_D) that satisfies this condition. The best of these commodity bundles is characterized by a tangency between one

of her indifference curves and the budget line. This tangency is described mathematically by the condition:

$$MRS_D = p \tag{2}$$

Applying the usual rule shows that:

$$MRS_D = \frac{\partial U_D}{\partial a_D} \div \frac{\partial U_D}{\partial b_D} = \frac{1}{2}\left(\frac{1}{a_D}\right)^{1/2}$$

Substituting this expression into (2) and simplifying yields Dinah's optimal consumption of ale:

$$a_D^* = \frac{1}{4}\left(\frac{1}{p}\right)^2 \tag{3}$$

Substituting this quantity into (1) yields her optimal bread consumption:

$$b_D^* = 10 - \frac{1}{4}\left(\frac{1}{p}\right)$$

Dinah's excess demand for ale is the difference between the quantity of ale that she wants and the quantity with which she is endowed. Since she is endowed with no ale, her excess demand for ale is:

$$ED_D = a_D^* - 0 = \frac{1}{4}\left(\frac{1}{p}\right)^2$$

b) Since Joe is endowed with no bread, he can only obtain bread by selling ale. If he wishes to consume a_J pints of ale, he can sell $18 - a_J$ pints, for which he will receive $p(18 - a_J)$ loaves of bread. Joe's budget constraint is therefore:

$$b_J = p(18 - a_J) \tag{4}$$

Joe's best attainable commodity bundle also satisfies the condition:

$$MRS_J = p$$

or equivalently:

$$\frac{1}{2}\left(\frac{1}{a_J}\right)^{1/2} = p \tag{5}$$

Joe's best attainable commodity bundle (a_J^*, b_J^*) is found by solving (4) and (5). The solution is:

$$a_J^* = \frac{1}{4}\left(\frac{1}{p}\right)^2$$

$$b_J^* = 18p - \frac{1}{4}\left(\frac{1}{p}\right)$$

Joe's excess demand for ale is the difference between the quantity of ale that he wants and the quantity of ale with which he was endowed:

$$ED_J = a_J^* - 18 = \frac{1}{4}\left(\frac{1}{p}\right)^2 - 18$$

c) The market clears when the sum of excess demands is equal to zero:

$$ED_D + ED_J = 0$$

$$\frac{1}{2}\left(\frac{1}{p}\right)^2 - 18 = 0$$

$$\left(\frac{1}{p}\right)^2 = 36$$

$$p = \frac{1}{6}$$

At this price, Dinah's excess demand for ale is 9, so she sells 3/2 loaves of bread to obtain 9 pints of ale. Joe's excess demand is -9, so he sells 9 pints of ale for 3/2 loaves of bread. The markets are clearing. To find the commodity bundles that Dinah and Joe actually consume, evaluate their optimal commodity bundles at the market-clearing price:

$$(a_D^*, b_D^*) = (9, 17/2)$$

$$(a_J^*, b_J^*) = (9, 3/2)$$

Note that Dinah and Joe together are consuming the economy's endowment of goods, 18 pints of ale and 10 loaves of bread.

Question 4 a) Pierre has two sources of cheese: he is endowed with some, and he can acquire more by selling wine. He is endowed with 4 kilograms of cheese. He is also endowed with 8 bottles of wine, and if he chooses to consume only w bottles, he is able to sell $8 - w$ bottles. Each bottle sells for p kilograms of cheese, so the total amount of cheese that he obtains from this source is $p(8-w)$. The total quantity of cheese that he consumes is the sum of these amounts:

$$c = 4 + p(8 - w) \tag{6}$$

This equation is his budget constraint, describing the commodity bundles that he can consume. The best of these commodity bundles is the one at which one of his indifference curves is tangent to the budget line. This tangency is expressed by the condition:

$$MRS = p$$

This condition argues that two rates are equal: the rate at which he can trade bread for cheese on the market, and the rate at which he is just willing to trade bread for cheese. Since p

represents the amount of cheese that he obtains by giving up one bottle of wine, MRS must be the amount of cheese that exactly compensates him for giving up one wine. Then:

$$MRS = \frac{\text{value of good given up}}{\text{value of each unit of the good given in compensation}} = \frac{\partial U}{\partial w} \div \frac{\partial U}{\partial c} = 3\left(\frac{c}{w}\right)$$

Thus, the tangency condition is:

$$3\left(\frac{c}{w}\right) = p \tag{7}$$

Pierre's optimal commodity bundle is found by solving (6) and (7):

$$c^* = 1 + 2p$$

$$w^* = \frac{3}{p} + 6$$

b) The quantity of wine that Pierre would like to buy ($i.e.$, his excess demand) is the difference between the quantity that he wants and the quantity that he has:

$$ED_P = w^* - 8 = \frac{3}{p} - 2$$

c) The market clears when the sum of the excess demands is equal to zero:

$$ED_P + ED_G = 0$$

$$\frac{7}{p} = 4$$

$$p = \frac{7}{4}$$

Chapter 4: The Production Economy

Question 1

a) Both production functions are Cobb-Douglas. Applying the rule for quick Cobb-Douglas derivatives and simplifying gives:

$$MP_K^a = \frac{1}{2}\left(\frac{L_a}{K_a}\right)^{1/2}$$

$$MP_L^a = \frac{1}{2}\left(\frac{K_a}{L_a}\right)^{1/2}$$

$$MP_K^b = \frac{1}{3}\left(\frac{L_b}{K_b}\right)^{2/3}$$

$$MP_L^b = \frac{2}{3}\left(\frac{K_b}{L_b}\right)^{1/3}$$

b) The marginal products are:

$$MP_K^a = 1, \quad MP_L^a = \frac{1}{4}, \quad MP_K^b = 3, \quad MP_L^b = \frac{2}{9}$$

i) Moving one unit of capital out of the ale industry reduces ale production by 1 unit, so moving 1 unit of capital out of the ale industry reduces ale production by one unit. This amount of capital, put to work in the bread industry, increases bread production by 3 units.

ii) Moving one unit of labour out of the ale industry reduces ale production by 1/4 units, so moving 4 units of labour out of the ale industry reduces ale production by one unit. This amount of labour, put to work in the bread industry, increases bread production by 8/9 units.

iii) If bread production is to be increased without any accompanying reduction in ale production, capital should be moved out of the ale industry and into the bread industry, and labour should be moved out of the bread industry and into the ale industry. Moving 1 unit of capital from the ale industry to the bread industry and 4 units of labour from the bread industry to the ale industry leaves ale production unchanged, because the first transfer reduces ale production by one unit and the second transfer raises ale production by one unit. The same pair of transfers increases bread production by 19/9 units.

c) The marginal products are now:

$$MP_K^a = \frac{1}{2\sqrt{2}}, \quad MP_L^a = \frac{1}{\sqrt{2}}, \quad MP_K^b = \frac{1}{3}, \quad MP_L^b = \frac{2}{3}$$

Suppose that capital is moved from the ale industry to the bread industry, and labour is moved in the opposite direction, so that ale production does not change. What transfers would hold ale production constant, and what would be their effect on bread production?

Let's look for movements of capital and labour that *individually* change ale production by one unit. Moving one unit of capital out of the ale industry reduces ale production by $1/2\sqrt{2}$, so $2\sqrt{2}$ units of capital must be moved to reduce ale production by one unit. This quantity of capital, when moved into the bread industry, raises bread production by $(2\sqrt{2}) \times (1/3)$ units. Moving one unit of labour into the ale industry increases ale production by $1/\sqrt{2}$, so $\sqrt{2}$ units of labour must be moved to increase ale production by one unit. Moving this amount of labour out of the bread industry reduces bread production by $(\sqrt{2}) \times (2/3)$ units. Thus, movements of capital and labour between the two industries that leave ale production unchanged will also leave bread production unchanged. Since there is no way of juggling the allocation of factors so that bread production can be increased without reducing ale production, the economy is operating on its production possibility frontier.

Question 2

a) An allocation determines George's and Harriet's commodity bundles, so the four variables are a_G, b_G, a_H, and b_H.

b) In this case, the requirements for Pareto optimality are that the allocation be exchange

efficient and match efficient, and that it correspond to a point on the production possibility frontier. These three requirements can be expressed as:

$$MRS_G = MRS_H$$

$$MRS = MRT$$

$$b_G + b_H = 2(10 - a_G - a_H)^{1/2} \tag{1}$$

(In the second condition, MRS is the common value of MRS_G and MRS_H. The third condition incorporates the fact that every unit of goods produced must be given to either George or Harriet.) There are infinitely many allocations that satisfy these conditions; the allocation that we are looking for also satisfies the condition:

$$U_H = 8$$

These conditions must be rewritten as equations that contain only the four variables that we wish to determine.

First condition: Use the rule for evaluating marginal rates of substitution:

$$MRS_G = \frac{\partial U_G}{\partial a_G} \div \frac{\partial U_G}{\partial b_G} = \frac{b_G}{a_G}$$

$$MRS_H = \frac{\partial U_H}{\partial a_H} \div \frac{\partial U_H}{\partial b_H} = \frac{1}{2}$$

Substituting these expressions into the exchange efficiency condition gives:

$$\frac{b_G}{a_G} = \frac{1}{2}$$

or:

$$a_G = 2b_G \tag{2}$$

Second condition: It is evident that the common value of the marginal rates of substitution must be $1/2$, so this value can be substituted into the left-hand side of the second condition. To evaluate the right-hand side of this condition, recall that the marginal rate of transformation is the number of units of bread that must be given up to get an ale, so:

$$MRT = -\frac{\partial b}{\partial a} = (10 - a_G - a_H)^{-1/2}$$

Thus, the second condition is:

$$\frac{1}{2} = (10 - a_G - a_H)^{-1/2}$$

This condition can be written much more simply:

$$\frac{1}{2} = \left(\frac{1}{10 - a_G - a_H}\right)^{1/2}$$

$$\frac{1}{4} = \frac{1}{10 - a_G - a_H}$$

$$a_G + a_H = 6 \tag{3}$$

Third condition: This condition is already in the required form.
Fourth condition: Substituting Harriet's utility function into this condition gives:

$$a_H + 2b_H = 8 \tag{4}$$

c) The required Pareto optimal allocation is found by solving the equation system (1)–(4). Any way of solving this system is fine, but here's one way to do it. First, note that (3) can be substituted into (1) to obtain a much simpler equation:

$$b_G + b_H = 4 \tag{5}$$

The equation system is now (2)–(5), which is readily solved by recursive substitution. Use (2) and (4) to eliminate a_G and a_H from (3):

$$b_H - b_G = 1 \tag{6}$$

Now (5) and (6) constitute a very simple system. Its solution is:

$$b_G^* = 3/2$$

$$b_H^* = 5/2$$

Substituting these values back into (2) and (4) yields:

$$a_G^* = 3$$

$$a_H^* = 3$$

Question 3

a) The four variables are again a_G, b_G, a_H, and b_H.

b) Again, there are three conditions that must be satisfied by any Pareto optimal allocation, and a fourth which identifies a particular allocation.

$$MRS_G = MRS_H$$

$$MRS = MRT$$

$$b_G + b_H = 10 - a_G - a_H \tag{7}$$

$$a_G = \widetilde{a} \tag{8}$$

The last two are already in the required form, but the first two must be rewritten so that they contain only the four variables that we wish to find. After the marginal rates of substitution and the marginal rate of transformation have been evaluated, these conditions are:

$$2\left(\frac{b_G}{a_G}\right) = \frac{b_H}{a_H} \tag{9}$$

$$\frac{b_H}{a_H} = 1 \tag{10}$$

Thus, the required equation system is (7)–(10).

c) Here is one way of solving this system. Combining (9) and (10) yields:

$$a_G = 2b_G$$

Use this equation and (10) to eliminate b_G and b_H from (7), and then use (8) to eliminate a_G from the resulting equation. This equation determines a_H. Since a_G is already known—it is $\widetilde{a}$—the values of a_G and a_H can then be used to determine b_G and b_H. The allocation is:

$$a_G^* = \widetilde{a}$$

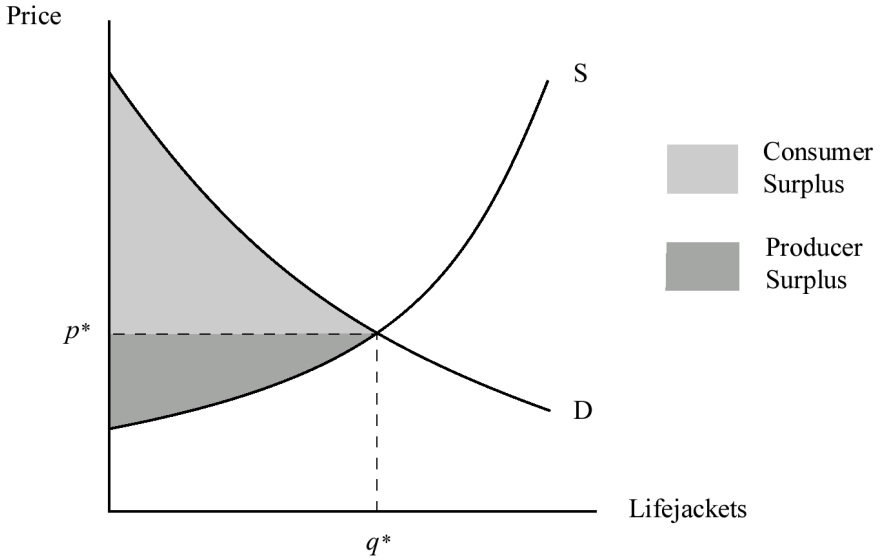
$$b_G^* = \frac{\widetilde{a}}{2}$$

$$a_H^* = b_H^* = 5 - \left(\frac{3}{4}\right)\widetilde{a}$$

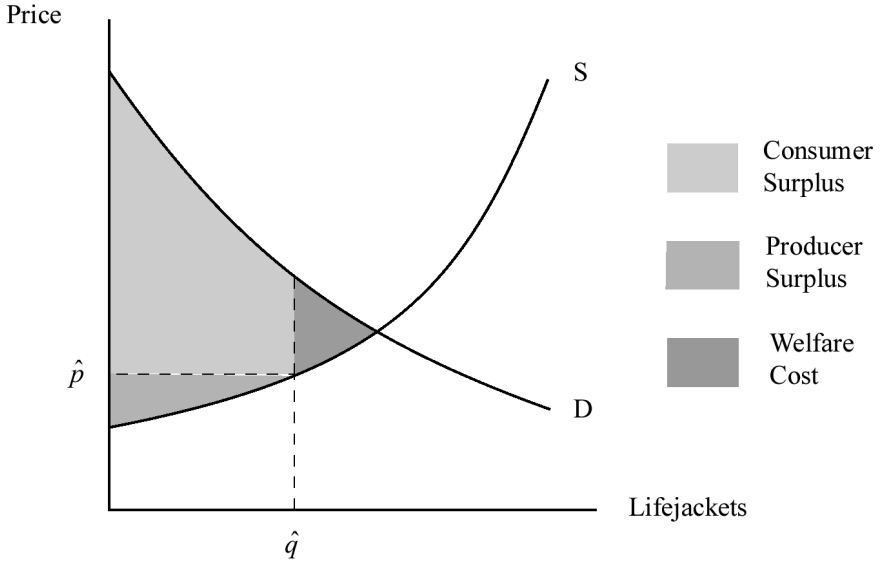
Chapter 5: Consumer and Producer Surplus

Question 1

The figure below shows the market before the government intervenes. Consumer surplus is the value to consumers of q^* lifejackets less the amount paid for them. The value of the lifejackets to the consumers is the area under the demand curve to the left of q^*, and the amount paid for them is the area of the rectangle with height p^* and width q^*. Producer surplus is the difference between the amount that firms are paid for the lifejackets less the smallest amount that the firms would have been willing to accept. The amount paid is the area of the rectangle, because in the absence of taxes or subsidies, the amount paid by consumers is equal to the amount received by firms. The smallest amount that the firms would have been willing to accept is the area under the supply curve to the left of q^*.



The next figure shows the market after the imposition of the price ceiling. At the price $\widehat{p}$, the firms are only willing to sell $\widehat{q}$ lifejackets. The amount paid for these lifejackets by consumers, and the amount received by firms, is the area of the rectangle with height $\widehat{p}$ and width $\widehat{q}$. Consumer surplus is the area under the demand curve to the left of $\widehat{q}$, less the area of the rectangle. Producer surplus is the area of the rectangle less the area under the supply curve to the left of $\widehat{q}$. It is evident that producer surplus has fallen, but it is not clear whether consumer surplus has risen or fallen. Surplus—the sum of consumer and producer surplus—has fallen, and the surplus lost is the welfare cost of this intervention.



Question 2 a) The equations for the supply and demand curves contain four variables, but four equations are needed to determine four variables. The "missing" equations describe the relationships between the two prices and the two quantities. In the absence of a tax or subsidy, the price paid by the buyer is equal to the price received by the seller:

$$p^D = p^S$$

Also, prices will adjust to ensure that the market clears—that the quantity demanded by the buyers is just equal to the quantity offered for sale by the sellers:

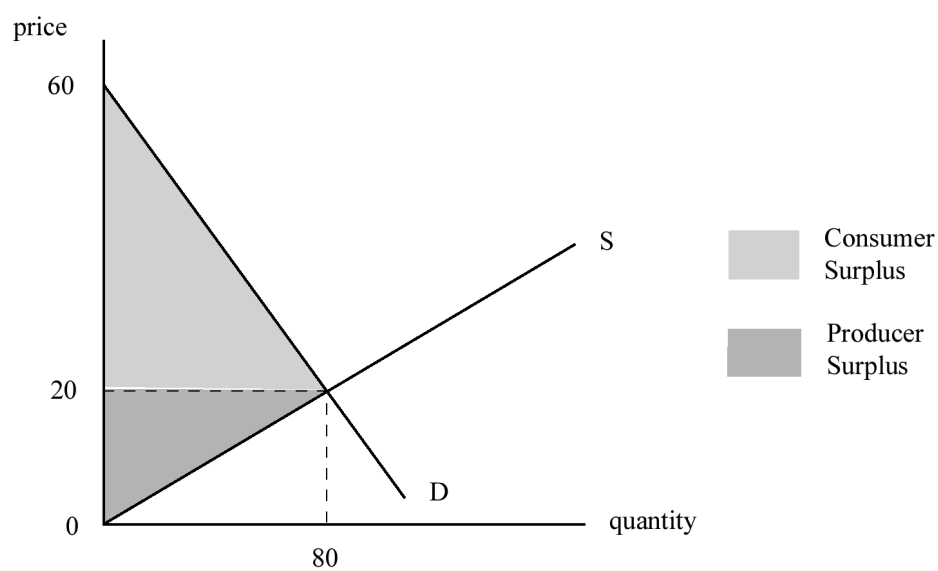
$$q^D = q^S$$

Let p be the common value of p^D and p^S, and let q be the common value of q^D and q^S. Then the system reduces to the two equation system:

$$q = 120 - 2p$$

$$q = 4p$$

Solving this system yields the equilibrium price (20) and quantity (80). This equilibrium is shown in the figure below.



Consumer and producer surplus can be calculated by recalling that the area of a triangle is equal to one-half of the product of its height and width. Consumer surplus is $\$(1/2)(60 - 20)(80)$, or \$1600. Producer surplus is $\$(1/2)(20)(80)$, or \$800. (Note that quantity is measured in physical units, while price is measured in dollars *per unit*. The product of price and quantity is therefore measured in dollars.)

b) The impositions of a tax pushes the price paid by the buyer above the price received by the seller. In the case at hand, \$6 of the price paid for each unit of goods by the buyers ends

up in the government's pockets instead of the sellers' pockets. This observation is expressed algebraically by the equation:

$$p^S = p^D - 6$$

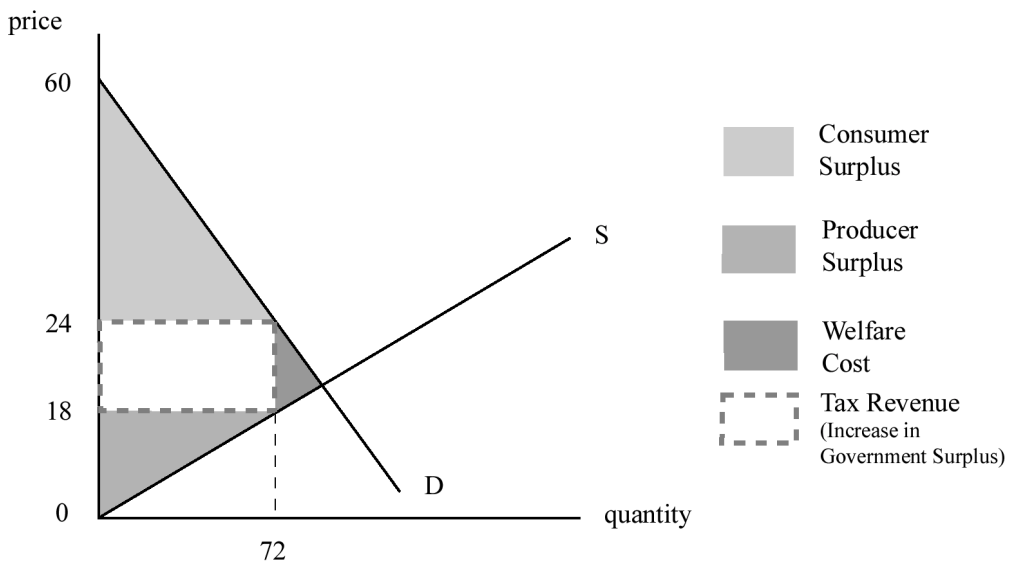
Nothing else changes: the behaviours of the buyers and sellers are still represented by the demand and supply equations, and the market will still clear. Letting q be the common value of q^D and q^S, the new equation system is:

$$q = 120 - 2p^D$$

$$q = 4p^S$$

$$p^S = p^D - 6$$

Solving this system shows that, in the presence of the tax, only 72 units of the good are exchanged. The price paid by the buyer \$24, and the price received by the seller is only \$18. The new equilibrium is shown in the next figure.



The total amount that buyers pay for the 72 units of goods is equal to the area of the rectangle with height 24 and width 72. The total amount that sellers receive is the area of the rectangle with height 18 and width 72. The difference between the areas of these two rectangles (*i.e.*, the area of the white rectangle) is the tax revenue collected by the government.

Consumer surplus is the area under the demand curve to the left of 72 less the amount paid by the buyers. It is equal to $\$(1/2)(60 - 24)(72)$, or \$1296. Producer surplus is the amount received by sellers less the area under the supply curve to the left of 72. It is $\$(1/2)(18)(72)$, or \$648. Together, the buyers and the sellers have lost \$456 of surplus. Not all of this surplus has been lost to the economy: the government has appropriated \$(6)(72), or \$432, of surplus

in the form of tax revenue. The remaining \$24 dollars of surplus is entirely lost, and it is the welfare cost of the tax. (Note that this welfare cost could have been calculated directly as the area of a (sideways) triangle with width $24 - 18$ and height $80 - 72$.)

c) When the government subsidizes a good, it pays part of the purchase price for the buyer. Together, the government and the buyer pay the amount demanded by the seller. In the present case, in which the government's share is \$3,

$$p^S = p^D + 3$$

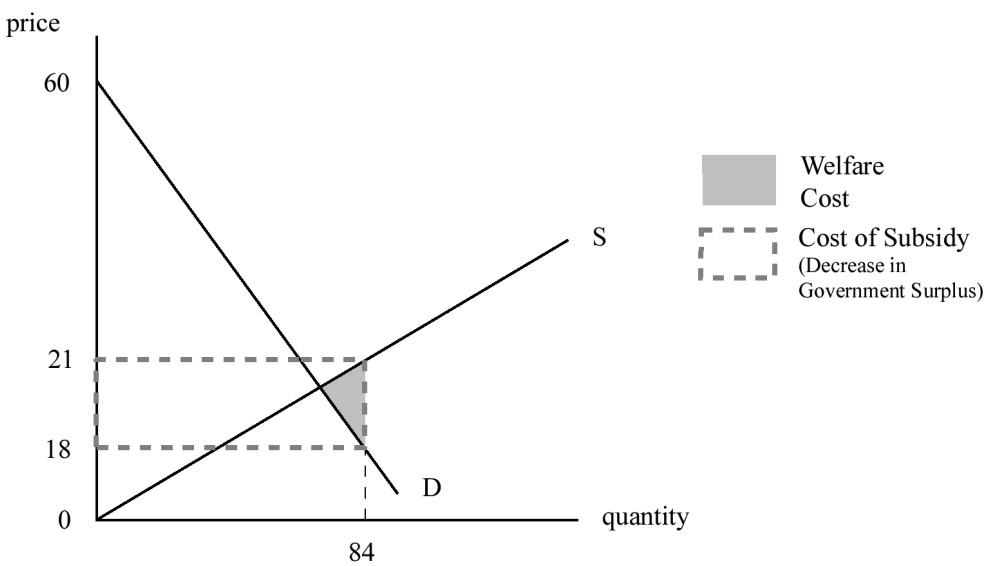
The system is now:

$$q = 120 - 2p^D$$

$$q = 4p^S$$

$$p^S = p^D + 3$$

Solving this system shows that, in the new equilibrium, 84 units are exchanged. The price received by the seller is \$21 while the price paid by the buyer is \$18. This policy costs the government \$(3)(84), or \$252. The equilibrium is shown in the figure below.



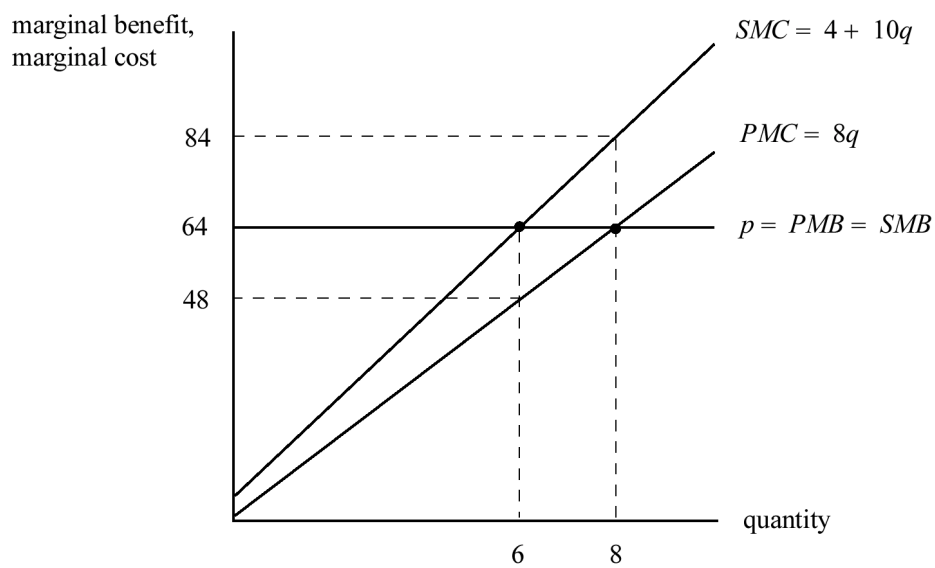
As compared to the initial (no intervention) equilibrium, this policy *increases* both consumer surplus and producer surplus. Consumer surplus is the area under the demand curve to the left of 84, less the area of the rectangle with height 18 and width 84. It is now $\$(1/2)(60 - 18)(84)$, or \$1764. Producer surplus is the area of the rectangle with height 21 and width 84, less the area under the supply curve to the left of 84. It is now $\$(1/2)(21)(84)$, or \$882.

In the absence of intervention, consumer surplus was only \$1600 and producer surplus

was only \$800. Thus, the subsidy increased the sum of consumer and producer surplus by \$246. Since the government had to give up \$252 of surplus to implement the subsidy, the program resulted in the loss of \$6 of surplus. This lost surplus is the welfare cost of the subsidy.

Graphically, the increase in the sum of consumer and producer surplus is equal to the area of the broken rectangle less the shaded area. The cost of the program is equal to the area of the broken rectangle, so the welfare cost of the program is the area of the shaded triangle: $\$(1/2)(21-18)(84-80)$.

Chapter 6: Externalities and Negotiation



a) If the firm has the property rights and there is no agreement, the firm will choose the level of output that maximizes its profits π, which are equal to the difference between its revenue from the sale of goods and its private costs. The profit-maximizing level of output is characterized by the condition:

$$PMC = PMB$$

where:

$$PMC = \frac{dc}{dq} = 8q$$

$$PMB = p = 64$$

Thus, the firm produces 8 units of output. Negotiations will continue until the firm's output

has been reduced to the optimal level, which is characterized by the condition:

$$SMC = SMB$$

Since:

$$SMC = PMC + MD = \frac{dc}{dq} + \frac{dD}{dq} = 4 + 10q$$

$$SMB = p = 64$$

the optimal level of output is 6. (Negotiations cease once this output level has been reached because, at this output, an additional small reduction in output reduces the neighbours' damages and the firm's profits by the same amount. The neighbours' benefit from that reduction is the difference between the reduced damage and the additional compensation that must be paid to the firm. Since this amount is not positive, the neighbours have no further interest in negotiating a further reduction in output.)

The smallest possible bribe just compensates the firm for its lost profits. Since the vertical distance between p and PMC is the firm's marginal profits, the reduction in total profits when output is reduced from 8 to 6 is the area between the p and PMC curves, and between the output levels 6 and 8. This area is readily calculated using the formula for the area of a triangle:

$$\text{minimum bribe} = (1/2)(64 - 48)(8 - 6) = 16$$

The largest possible bribe is the reduction in damages associated with the reduction in output. Marginal damage is the vertical distance between PMC and SMC, so the reduction in damages is the area between these curves, and between the output levels 6 and 8. It is the sum of the areas of two triangles:

$$\text{maximum bribe} = (1/2)(64 - 48)(8 - 6) + (1/2)(84 - 64)(8 - 6) = 36$$

b) If the neighbours have the property rights and there is no agreement, the neighbours will avoid all damages by refusing to allow the firm to produce. Negotiations will take the output level to the optimal level. (Once this level has been reached, the compensation that the firm must pay to the neighbours to get a marginal increase in output is at least as big as the increase in profits generated by that increase. There would be no benefit to the firm from negotiating such an increase.)

The smallest bribe compensates the neighbours for the damage inflicted upon them. It could be calculated as an area, as it was above, but it can also be calculated algebraically:

$$\text{minimum bribe} = D|_{q=6} - D|_{q=0} = 60$$

The largest bribe is equal to the increase in the firm's profits when its output rises from 0 to 6:

$$\text{maximum bribe} = \pi|_{q=6} - \pi|_{q=0} = \left[(64)(6) - 4(6)^2\right] - 0 = 240$$

c) The ideal tax forces the firm to recognize the damages that its activities imposes upon the neighbours, and is therefore equal to marginal damage evaluated at the optimal level of

output:

$$MD|_{q=6} = 16$$

The imposition of this tax will shift the firm's private marginal cost curve upward so that it intersects the price line at the optimal level of output, inducing the firm to produce at this level. The government's revenue is the product of the unit tax and the number of units (16×6).

Chapter 7: Permit Trading

Question 1

Abatement costs under direct emissions controls are:

$$(c_1 + c_2)_d = \left(\frac{m_1 + m_2}{2}\right)(\overline{x} - \widehat{x})^2$$

Abatement costs under permit trading differ in the two cases described in the text. Let's consider these cases in turn.

Case 1. In this case, abatement costs are:

$$(c_1 + c_2)_p = 2\,(\overline{x} - \widehat{x})^2 \left(\frac{1}{m_1} + \frac{1}{m_2}\right)^{-1}$$

Permit trading is cheaper if:

$$\frac{m_1 + m_2}{2} > 2\left(\frac{1}{m_1} + \frac{1}{m_2}\right)^{-1}$$

Simplifying this expression yields:

$$(m_1 - m_2)^2 > 0$$

which necessarily holds.

Case 2. Abatement costs in this case can be expressed as:

$$(c_1 + c_2)_p = (c_1 + c_2)_d + \frac{\widehat{x}}{2}\left[m_1(2\overline{x} - \widehat{x}) - m_2(2\overline{x} - 3\widehat{x})\right]$$

so that permit trading is cheaper if:

$$m_1(2\overline{x} - \widehat{x}) < m_2(2\overline{x} - 3\widehat{x})$$

or equivalently,

$$\widehat{x} < \overline{x}\left[\frac{2(m_2 - m_1)}{3m_2 - m_1}\right] \tag{1}$$

The expression in square brackets is a positive fraction, so if $\widehat{x}$ is close to $\overline{x}$, this inequality will not be satisfied. However, case 2 describes only situations in which $\widehat{x}$ is *relatively far* from $\overline{x}$. It turns out that if $\widehat{x}$ is far enough from $\overline{x}$ for case 2 to apply, it is far enough from $\overline{x}$

for this inequality to hold. To see this, recall that case 2 requires:

$$\widehat{x} < \overline{x}\left[\frac{m_2 - m_1}{2m_2}\right]$$

Since:

$$\frac{m_2 - m_1}{2m_2} < \frac{2(m_2 - m_1)}{3m_2 - m_1}$$

the inequality (1) holds for all parameter values consistent with case 2.

Question 2

a) Firm 1 has $\widehat{x}$ permits and must possess a permit for each unit of emissions. It must therefore purchase $(1/2)(q_1)^2 - \widehat{x}$ permits. Recall that a negative purchase is to be interpreted as a sale.

b) Firm 1's profits are:

$$\pi_1 = \theta q_1 - p\left[\left(\frac{1}{2}\right)(q_1)^2 - \widehat{x}\right]$$

If there were no permit trading, the firm would earn profit θ on each of q_1 units of output. With permit trading in place, it must also buy each of the required permits at the price p. This additional cost reduces the firm's profits.

c) The profit-maximizing output is found by calculating the required derivative and setting it equal to zero:

$$\frac{d\pi_1}{dq_1} = \theta - pq_1 = 0$$

$$\overset{\circ}{q_1} = \theta/p$$

Substituting the profit-maximizing output into the expression for the firm's purchases of permits yields its excess demand for permits:

$$ED_1 = e_1 - \widehat{x} = \left(\frac{1}{2}\right)(\overset{\circ}{q_1})^2 - \widehat{x} = \left(\frac{1}{2}\right)\left(\frac{\theta}{p}\right)^2 - \widehat{x}$$

d) Firm 2's profits are:

$$\pi_2 = \theta q_2 - p\left[\left(\frac{1}{4}\right)(q_2)^2 - \widehat{x}\right]$$

Calculating the required derivative and setting it equal to zero yields firm 2's profit-maximizing level of output:

$$\theta - \left(\frac{1}{2}\right)q_2 = 0$$

$$\overset{\circ}{q_2} = \frac{2\theta}{p}$$

Firm 2's excess demand for permits is:

$$ED_2 = \left(\frac{1}{4}\right)(\overset{\circ}{q_2})^2 - \widehat{x} = \left(\frac{\theta}{p}\right)^2 - \widehat{x}$$

e) Both of the excess demands become larger as p falls. Firm 1's excess demand is equal to zero when:

$$p = \frac{\theta}{\sqrt{2\widehat{x}}}$$

and firm 2's excess demand is equal to zero when:

$$p = \frac{\theta}{\sqrt{\widehat{x}}}$$

Since the latter price is greater than the former price, there are no prices at which firm 1's excess demand is positive and firm 2's excess demand is negative—that is, there are no prices at which firm 1 would want to buy while firm 2 would want to sell. However, firm 2 will want to buy permits and firm 1 will want to sell permits at all prices in the range:

$$\frac{\theta}{\sqrt{2\widehat{x}}} < p < \frac{\theta}{\sqrt{\widehat{x}}}$$

The market-clearing price must lie within this range.

f) The market clears when the sum of excess demands is equal to zero:

$$ED_1 + ED_2 = 0$$

$$\left(\frac{3}{2}\right)\left(\frac{\theta}{p}\right)^2 = 2\widehat{x}$$

$$\overset{\circ}{p} = \frac{\theta}{2}\sqrt{\frac{3}{\widehat{x}}}$$

Evaluating the excess demands at this price—which is the market-clearing price—shows that firm 1 sells $\widehat{x}/3$ permits to firm 2. After the trading, firm 1 will have $(2/3)\widehat{x}$ permits and firm 2 will have $(4/3)\widehat{x}$ permits. Output can be calculated by noting that each firm's emissions e will be equal to the number of permits that it holds. However, it is perhaps simpler to substitute the market-clearing price into the expressions for the profit-maximizing outputs. It is found that:

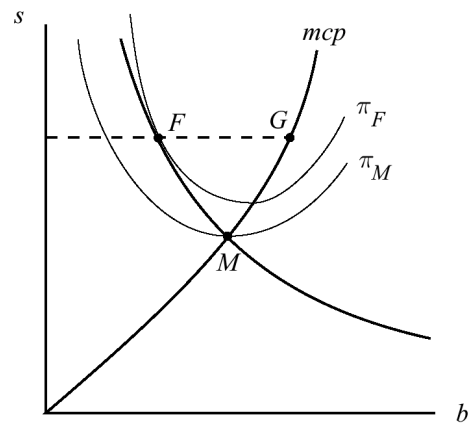
$$\overset{\circ}{q_1} = 2\sqrt{\frac{\widehat{x}}{3}}$$

$$\overset{\circ}{q_2} = 4\sqrt{\frac{\widehat{x}}{3}}$$

That is, firm 2 ends up with twice as many permits and produces twice as much output.

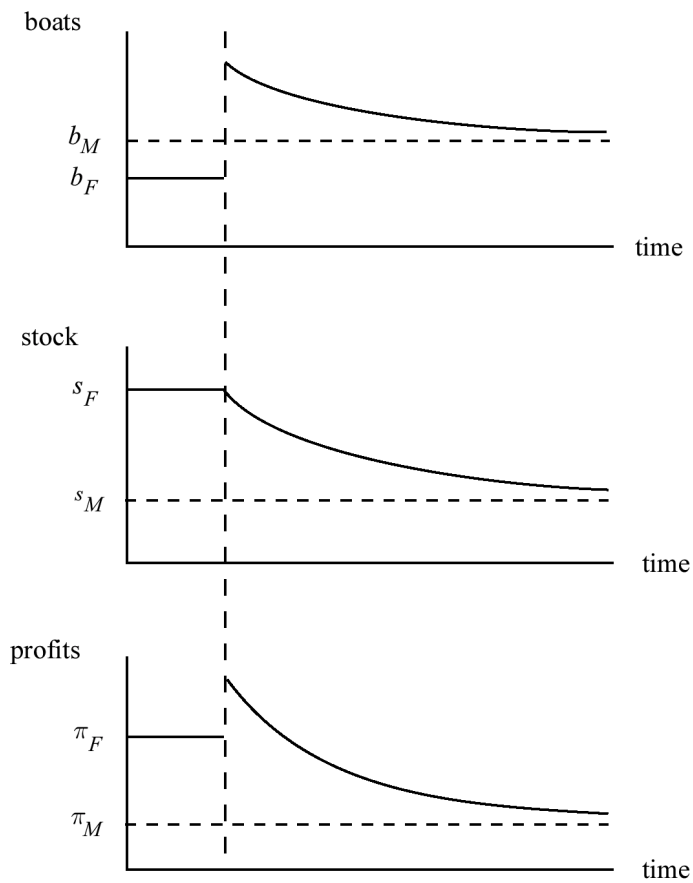
Chapter 8: Renewable Common Property Resources

Question 1



a) The replacement of the manager has no immediate impact on the stock, but the new manager increases the number of boats to reach the mcp curve. Consequently, the state of the fishery changes from F to G. Since G is to the right of the ss locus, the stock begins to fall. With each fall in the stock the myopic manager reduces the number of boats to remain on the mcp curve. Thus, the new steady state, M, is approached by sliding down the mcp curve. The consequences of these policies for b and s are shown in the top panels of the next figure. The stock falls continuously towards its new steady-state value, while the number of boats jumps up and then begins to fall, though it never falls as low as it was in the old steady state.

b) Profits in the old steady state were π_F. The movement from F to G increases the fishery's profits (since G lies on an iso-profit curve that lies above the π_F iso-profit curve). However, the subsequent adjustment of the fishery causes profits to drop continuously. The profits in the new steady state are π_M, where $\pi_M < \pi_F$. The time path of profits is shown in the bottom panel of the next figure.



Question 2 a) In the absence of hunting, the stock of mammoths will grow until net natural additions drop to zero. The steady-state stock is found by setting g equal to zero:

$$g = \frac{1}{50}\left(s - \frac{s^2}{50}\right) = 0$$

Simplifying gives:

$$s^* = 50$$

(The time subscripts have been omitted here and in the remainder of this problem, as only steady states are being considered.)

b) The steady state lies at the intersection of the ss and mcp curves, so the first step is to derive the equations for these two curves. The ss curve shows all of the pairs (h, s) that satisfy the condition:

$$g = y$$

Substituting into this equation and simplifying gives:

$$s = 50(1 - h) \tag{1}$$

The mcp curve shows all of the pairs (h, s) such that the amount of hunting activity maximizes current profits. These profits are:

$$\pi = y - c = \frac{hs}{50} - \frac{h^2}{5}$$

The profit-maximizing value of h satisfies the condition:

$$\frac{d\pi}{dh} = 0$$

(That is, there is no small adjustment in hunting activity that will raise profits.) Evaluating the derivative gives:

$$\frac{s}{50} - \frac{2h}{5} = 0$$

or:

$$s = 20h \tag{2}$$

Solving (1) and (2) shows that, in the steady state, the stock of mammoths is 100/7 and the amount of hunting is 5/7.

c) The new steady state must still satisfy (1)—that is, it must lie on the ss locus—but the rule for determining the amount of hunting has changed. Hunting now continues until profits are driven to zero:

$$\pi = y - c = 0$$

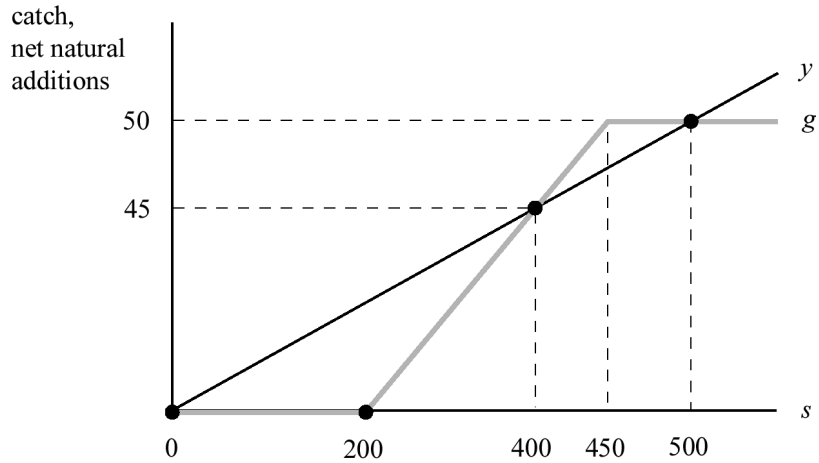
Substituting into this condition and simplifying yields:

$$s = 10h \tag{3}$$

The new steady state satisfies (1) and (3). In this steady state, the stock is 50/6 and the amount of hunting is 5/6.

Question 3

a) The graphs of y and g are shown below. They intersect twice in the interior of the graph.



b) The aardvark population is constant through time if catch is equal to net natural additions $(y = g)$.

c) Inspection of the figure above shows that the graph of net natural additions intersects the graph of catch at two positive values of s. To find the numerical values, note that one of these intersections occurs on the middle section of the g locus, so the condition $y = g$ becomes:

$$.1s = .2(s - 200)$$

Solving this condition shows that the aardvark population is in a steady state when there are 400 aardvarks. Inspection of the figure shows that there is an intersection in the final section of the net natural additions curve. The locus $y = g$ is then:

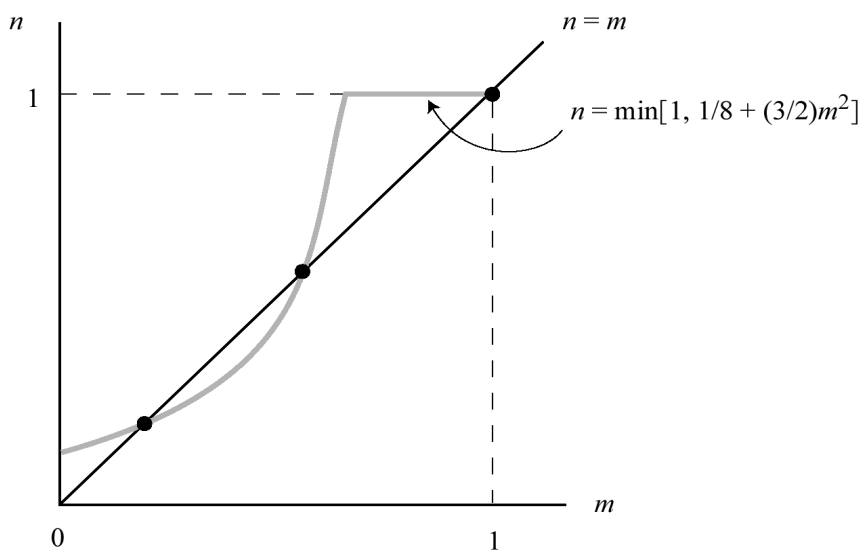
$$.1s = 50$$

implying a population of 500.

d) The aardvark population grows when g exceeds y and shrinks when y exceeds g. Inspection of the figure shows that, if the population initially exceeds 400, the steady state population will be 450. If the initial population is 400, the population will remain at 400. Finally, if the population is less than 400, catch will always exceed net natural additions, causing the population to shrink to nothing. Thus, $\widehat{s}$ is 400.

Chapter 9: Co-ordination Failures

Question 1



The figure above shows the graph of the relationship between n and m, as well as the graph of the equilibrium condition. The intersection points of the two graphs are equilibrium group sizes. One of the equilibrium group sizes is 1 (everyone participates). The remaining equilibrium group sizes satisfy the conditions:

$$n = \frac{1}{8} + \left(\frac{3}{2}\right) m^2$$

$$n = m$$

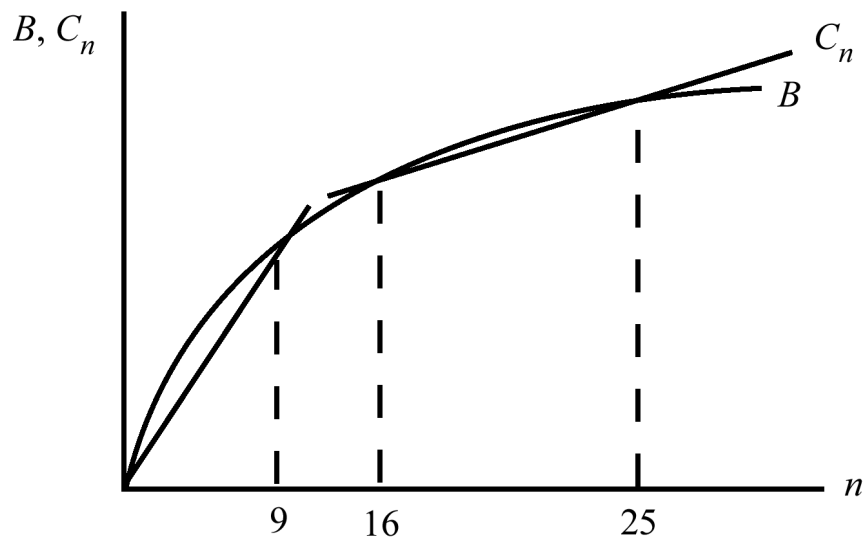
The first equation is the relationship between n and m. (The min function can be dropped because we know that we are looking for values of n that are less than one.) The second is the equilibrium condition. Substituting the second equation into the first yields the quadratic equation:

$$m = \frac{1}{8} + \left(\frac{3}{2}\right) m^2$$

Applying the quadratic rule shows that the two solutions to this equation are 1/6 and 1/2.

Question 2

Treat n as a continuous variable for the moment. Sketching the functions shows that B and C_n cross three times.



Let n^* satisfy the equation:

$$B = C_n$$

If n^* is an integer, it is potentially an equilibrium; and if it is not an integer, the largest integer below n^* is potentially an equilibrium. In fact, there are three such solutions and they are all integers: 9, 16 and 25. The diagram above shows that only 9 and 25 satisfy the second equilibrium conditions.

To obtain the values of n^*, first note that the smallest value satisfies the condition:

$$9n^{1/2} = 3n$$

The larger two values of n^* satisfy the equation:

$$9n^{1/2} = 20 + n$$

Squaring both sides of this equation converts it into a quadratic equation:

$$81n = 400 + 40n + n^2$$

This equation can be solved by applying the quadratic rule, or by noting that it can be rewritten as:

$$(n - 16)(n - 25) = 0$$

There are two equilibria: either 9 people attend or 25 people attend. To show that the second of these equilibria is better than the first, the well-being of several kinds of person must be considered. First, persons 1 through 9 attend the book swap under both equilibria. They prefer bigger meetings to smaller meetings because the size of the meeting does not affect their personal travel costs, but bigger meetings yield larger benefits than smaller ones. Second,

persons 10 through 24 attend only large meetings. Consequently, they get zero net benefits—no benefits and no costs—from small meetings but positive net benefits from large meetings. (Person 25 also attends only large meetings, but is indifferent between attending large meetings and not attending them. He is indifferent between the two meeting sizes because his net benefits are zero under each equilibrium.) Finally, people with travel costs higher than that of person 25 do not attend under either equilibrium, so their net benefits are also zero under both equilibria. In short, persons 1 through 24 benefit from larger meetings, and everyone else is indifferent.

Chapter 10: Pure Public Goods

Question 1 The optimal quantity of the public good is provided when the sum of the fifty people's marginal rates of substitution is equal to the marginal rate of transformation. Since there are twenty people of the first type and thirty people of the second type, this condition reduces to:

$$20MRS_1 + 30MRS_2 = MRT$$

$$(400 - 20z) + (900 - 60z) = 340 + 40z$$

$$z^* = 8$$

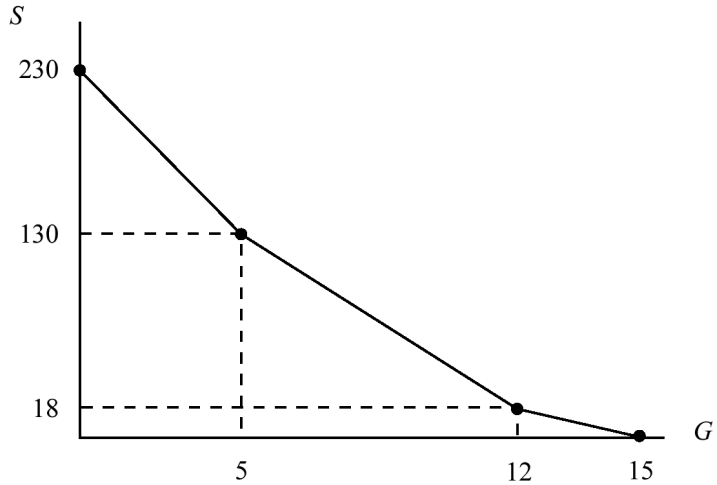
Question 2 The max functions make this problem messy, so let's begin by getting rid of them. Tom's marginal rate of substitution is positive when G is less than 5 and zero otherwise. Dick's marginal rate of substitution is positive when G is less than 15 and zero otherwise. Harry's marginal rate of substitution is positive when G is less than 12 and zero otherwise. Letting S be the sum of the marginal rates of substitution,

$$S = \begin{cases} (20 - 4G) + (90 - 6G) + (120 - 10G) & \text{if } 0 \leq G < 5 \\ (90 - 6G) + (120 - 10G) & \text{if } 5 \leq G < 12 \\ 90 - 6G & \text{if } 12 \leq G < 15 \\ 0 & \text{if } 15 \leq G \end{cases}$$

or more simply:

$$S = \begin{cases} 230 - 20G & \text{if } 0 \leq G < 5 \\ 210 - 16G & \text{if } 5 \leq G < 12 \\ 90 - 6G & \text{if } 12 \leq G < 15 \\ 0 & \text{if } 15 \leq G \end{cases}$$

Graphing S against G results in the following figure.



The Samuelson condition is:

$$S = MRT$$

If MRT is 150, inspection of the figure shows that S is equal to MRT when G is somewhere between 0 and 5. If G lies between 0 and 5,

$$S = 230 - 20G$$

so the Samuelson condition is:

$$230 - 20G = 150$$

This condition is solved to find the optimal quantity of public good, which is 4. Now suppose that MRT is 50. Inspection of the figure shows that S is equal to MRT when G is somewhere between 5 and 12, so:

$$S = 210 - 16G$$

The Samuelson condition is therefore:

$$210 - 16G = 50$$

and the optimal quantity of public good is 10.

Question 3 a) A Pareto optimal allocation is a triplet (b_G, b_H, z). Every Pareto optimal allocation lies on the production possibility frontier (because every unit of goods must be used somehow in a Pareto optimal allocation) and satisfies the Samuelson condition. A particular Pareto optimal allocation—the one in which George and Harriet have equal utilities—is being sought. Thus, the triplet is constrained by three equations: the production possibility frontier, the Samuelson condition, and the equal utility condition. Let's develop these equations in turn.

The frontier. The economy is endowed with 120 loaves of bread. They can be consumed, or they can be used to produce units of the public good. Since 20 loaves of bread are needed to produce each unit of public goods, the economy's production possibility frontier is:

$$b_G + b_H + 20z = 120 \tag{1}$$

The Samuelson condition. This condition states:

$$MRS_G + MRS_H = MRT$$

The marginal rate of transformation is the number of loaves of bread required to produce one unit of public goods. It is fixed at 20. The marginal rates of substitution are calculated in the usual fashion:

$$MRS_G = \frac{\partial U_G}{\partial z} \div \frac{\partial U_G}{\partial b_G} = \frac{b_G}{2z}$$

$$MRS_H = \frac{\partial U_H}{\partial z} \div \frac{\partial U_H}{\partial b_H} = \frac{b_H}{2z}$$

Substituting into the Samuelson condition gives:

$$\frac{b_G + b_H}{2z} = 20 \tag{2}$$

Equal utilities. This condition states that:

$$U_G = U_H$$

but since George and Harriet have the same utility function and consume the same quantity of the public good, this condition can also be written as:

$$b_G = b_H \tag{3}$$

The three equation system (1)—(3) determines the allocation. The solution to this system is:

$$b_G^* = 40, \quad b_H^* = 40, \quad z^* = 2$$

b) The contributions can be measured in either the public good or the private good. Assume that they are measured in the private good. George's consumption of bread is:

$$b_G = 70 - c_G \tag{4}$$

where c_G is his contribution. Similarly, Harriet's bread consumption is:

$$b_H = 50 - c_H \tag{5}$$

where c_H is her contribution. Since 20 loaves of bread are required to produce a unit of the public good, the quantity of that good provided will be:

$$z = \frac{c_G + c_H}{20} \tag{6}$$

(If the contributions were measured in units of the public good, z would be equal to the sum of the contributions, and each unit of contributions would reduce the contributor's bread consumption by 20 units.)

To find George's best response function, substitute (4) and (6) into his utility function:

$$U_G = \left(\frac{c_G + c_H}{20}\right)^{1/3} (70 - c_G)^{2/3}$$

George's utility-maximizing contribution satisfies the condition:

$$\frac{dU_G}{dc_G} = 0$$

Applying the rule for quick Cobb-Douglas derivatives and cleaning up a little gives:

$$\frac{U_G}{3}\left(\frac{1}{c_G + c_H} - \frac{2}{70 - c_G}\right) = 0$$

or:

$$c_G = \frac{70 - 2c_H}{3}$$

This is George's best response function. Similarly, Harriet's best response function is found by substituting (5) and (6) into her utility function, and then taking the derivative of this function with respect to c_H and setting it equal to zero. The resulting condition is:

$$\frac{U_H}{3}\left(\frac{1}{c_G + c_H} - \frac{2}{50 - c_H}\right) = 0$$

which is re-arranged to get Harriet's best response function:

$$c_H = \frac{50 - 2c_G}{3}$$

Note that the two reaction functions are *not* mirror images of each other (because George and Harriet don't have the same endowments) and hence the Nash equilibrium will not be symmetric.

The Nash equilibrium is a pair (c_G, c_H) that satisfies both best response functions—that is, it is the solution of a system of two equations containing two unknowns. Solving the system yields the Nash equilibrium contributions:

$$c_G^* = 22, \quad c_H^* = 2$$

These contributions are sufficient to product 6/5 units of the public good.

Question 4

a) Evaluating the derivatives shows that:

$$MRS_1 = \frac{1}{c_1}, \quad MRS_2 = \frac{1}{c_2}$$

b) Since the marginal rate of transformation is 1, the Samuelson condition is:

$$\frac{1}{c_1} + \frac{1}{c_2} = 1$$

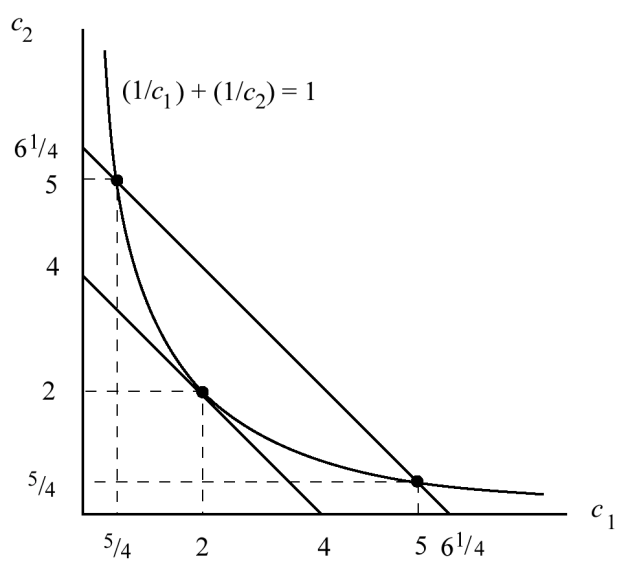
or equivalently:

$$c_2 = \frac{c_1}{c_1 - 1}$$

c) The figure below sketches the Samuelson condition and the adding-up condition for $s = 4$. It is readily verified that both of these curves pass through the point $(2, 2)$. Since the slope of both curves is equal to -1 at this point, they are tangent to each other at that point. Thus, $(2, 2)$ is the only solution to this pair of equations. That is, the only efficient way of producing 4 units of public goods is to obtain a contribution of 2 from each person.

If fewer than 4 units of the public good are to be produced, the adding-up condition lies closer to the origin. It does not intersect the Samuelson condition anywhere, and hence there are no efficient allocations in which fewer than 4 units of the public good are produced.

If more than 4 units of the public good are to be produced, the adding-up condition lies farther from the origin. It intersects the Samuelson condition twice, indicating that there are two efficient allocations that result in the production of this quantity of public goods. For example, if $6^1/_4$ units of public good are produced, the efficient allocations involve a contribution of 5 from person 1 and a contribution of $5/4$ from person 2, or a contribution of $5/4$ from person 1 and a contribution of 5 from person 2.



d) As s rises, the adding-up condition is pushed farther from the origin. The intersection points are pushed farther out along the arms of the Samuelson condition, so that (at each of the two solutions) one person's contribution rises while the other person's contribution falls.

Chapter 11: Two Examples of Public Goods

Question 1

a) Firm i's profits are:

$$\pi_i = 2\ln\left(x_i + (n-1)\widetilde{x}\right) - \frac{x_i}{2}$$

The profit-maximizing value of x_i satisfies the condition:

$$\frac{d\pi_i}{dx_i} = 0$$

Evaluating the derivative gives:

$$\frac{2}{x_i + (n-1)\widetilde{x}} = \frac{1}{2}$$

Re-arranging this equation gives firm i's best response function:

$$x_i = 4 - (n-1)\widetilde{x}$$

In a symmetric equilibrium, every firm is making a best response to the other firms' actions, and every firm's action is the same, implying:

$$x_i = \widetilde{x}$$

Thus, in a symmetric equilibrium, the stock of knowledge chosen by each firm is:

$$x = \frac{4}{n}$$

and the aggregate stock of knowledge is:

$$nx = 4$$

The profits of each firm are:

$$\pi = 2\ln 4 - \frac{2}{n}$$

b) Firm i's profits are:

$$\pi_i = 2\ln x_i - \frac{x_i}{2}$$

Its profits are maximized when:

$$\frac{2}{x_i} - \frac{1}{2} = 0$$

or equivalently,

$$x_i = 4$$

Each firm's stock of knowledge is independent of the stocks chosen by the other firm, so the symmetric equilibrium involves each firm choosing the stock of knowledge:

$$x = 4$$

leading to an aggregate stock of knowledge:

$$nx = 4n$$

Each firm's profits are:

$$\pi = 2\ln 4 - 2$$

c) Firm i's profits are:

$$\pi_i = \ln x_i + \ln\left(x_i + (n-1)\widetilde{x}\right) - \frac{x_i}{2}$$

Firm i's profits are maximized when it chooses a stock of knowledge satisfying the condition:

$$\frac{1}{x_i} + \frac{1}{x_i + (n-1)\widetilde{x}} = \frac{1}{2}$$

Simplifying this condition yields:

$$4x_i + 2(n-1)\widetilde{x} = (x_i)^2 + (n-1)x_i\widetilde{x}$$

which is a quadratic in x_i. The symmetric Nash equilibrium must satisfy this condition, and the requirement that every firm behave in the same way:

$$x_i = \widetilde{x}$$

Combining these conditions shows that, in the symmetric equilibrium, each firm chooses the stock:

$$x = 2\left(\frac{n+1}{n}\right)$$

leading to an aggregate stock of knowledge:

$$nx = 2(n+1)$$

Each firm's profits are:

$$\pi = \ln 2\left(\frac{n+1}{n}\right) + \ln 2(n+1) - \frac{n+1}{n}$$

d) The aggregate stock of knowledge is 4 when there are no patents, $2(n+1)$ when there are one-period patents, and $4n$ when there are two-period patents. A comparison of these values shows that the stock rises with the duration of patent protection if n is greater than 1.

e) The stock of knowledge employed by each firm is the same when there are no patents as when there are two-period patents. The difference between these cases is that the cost of making this amount of knowledge available to each firm when there are two-period patents

is n times as great as the cost when there are no patents. Profits are therefore greater when there are no patents.

f) Each firm's profits when there are no patents are 1.77, while each firm's profits when there are one-period profits are 1.39. Free-riding is not a great problem when there are very few firms.

g) Each firm's profits when there are no patents are 2.57, while each firm's profits when there are one-period profits are 2.78. Free-riding is a considerable problem when there are many firms, and one-period patents serve to reduce it.

Question 2

a) To find Tom's best response function, first note that:

$$c_T = y - z_T$$

$$c_H = z_T + z_D$$

Substituting these equations into Tom's utility function gives:

$$U_T = 2(y - z_T)^{1/2} + (z_T + z_D)^{1/2}$$

The contribution that maximizes his utility satisfies the condition:

$$\frac{dU_T}{dz_T} = 0$$

Evaluating the derivative and re-arranging gives:

$$(y - z_T)^{-1/2} = \left(\frac{1}{2}\right)(z_T + z_D)^{-1/2}$$

Squaring both sides yields:

$$\frac{1}{y - z_T} = \left(\frac{1}{4}\right)\frac{1}{z_T + z_D}$$

Simplifying this expression gives Tom's best response function:

$$z_T = \frac{y - 4z_D}{5}$$

Proceeding in a similar fashion gives Dick's best response function:

$$z_D = \frac{y - 4z_T}{5}$$

A Nash equilibrium is a pair of contributions that satisfies both best response functions. In the Nash equilibrium, Tom and Dick both contribute $y/9$.

b) If both people contribute the same amount, Tom and Dick both have the utility:

$$U = 2(y - z)^{1/2} + (2z)^{1/2}$$

The utility-maximizing value of z satisfies the condition:

$$(y-z)^{-1/2} = (2z)^{-1/2}$$

Squaring both sides and simplifying shows that the utility-maximizing value of z is $y/3$.

Chapter 12: Impure Public Goods

Question 1

The net benefits of the club are:

$$B - C = (36 - m^2)s^{2/3} - \frac{27s}{m}$$

The optimal membership and size are characterized by the conditions:

$$\frac{\partial(B-C)}{\partial m} = -2s^{2/3}m + \frac{27s}{m^2} = 0$$

$$\frac{\partial(B-C)}{\partial s} = (2/3)(36 - m^2)s^{-1/3} - \frac{27}{m} = 0$$

These two conditions can be expressed as:

$$s^{1/3} = \left(\frac{2}{27}\right) m^3 \tag{1}$$

$$s^{1/3} = \left(\frac{2}{3}\right)\left(\frac{m}{27}\right)(36 - m^2)$$

Combining these equations to eliminate s yields:

$$3m^2 = 36 - m^2$$

implying:

$$m^* = 3$$

Substitution into (1) yields:

$$s^* = 8$$

Question 2

The net benefits are:

$$B - C = s^{1/2}(20m - m^2 + 225) - \frac{\gamma s}{m}$$

The optimal membership and size satisfy the conditions:

$$\frac{\partial(B-C)}{\partial m} = s^{1/2}(20 - 2m) + \frac{\gamma s}{m^2} = 0$$

$$\frac{\partial(B-C)}{\partial s} = (1/2)s^{-1/2}(20m - m^2 + 225) - \frac{\gamma}{m} = 0$$

Re-writing these equations gives:

$$s^{1/2} = m^2(2m - 20)/\gamma$$

$$s^{1/2} = m(20m - m^2 + 225)/2\gamma$$

Combining these equations to eliminate $s^{1/2}$ and simplifying gives:

$$m^2 - 12m - 45 = 0$$

$$(m - 15)(m + 3) = 0$$

implying that the optimal membership is 15 (a membership of -3 doesn't make sense) for every γ. Setting γ equal to 1000 and m equal to 15 in one of the optimality conditions determines the optimal size, which is $81/16$.

Question 3

a) Substitution gives:

$$t^{\circ} = 4s^{2/3}$$

b) Substituting for t and v (which is equal to t) in the expression for social net benefits gives:

$$SNB = 288s^{1/3} - 144s^{1/3} - 3s = 144s^{1/3} - 3s$$

The optimal capacity satisfies the condition:

$$\frac{\partial SNB}{\partial s} = 48s^{-2/3} - 3 = 0$$

implying that the optimal size is 64.

c) The optimal toll satisfies the condition:

$$p = 72t^{-1/2} - (9t/s)$$

Substituting for t and s shows that the required toll is $15^3/_4$.

Chapter 13: The Link Between Public Goods and Externalities

Question 1

a) The marginal rates of substitution are:

$$MRS_D = \frac{1}{2h_D}$$

$$MRS_J = \frac{1}{h_J}$$

and the marginal rate of transformation is:

$$MRT = \frac{\partial c}{\partial h_D} = \frac{\partial c}{\partial h_J} = 1$$

so the Samuelson condition can be written as:

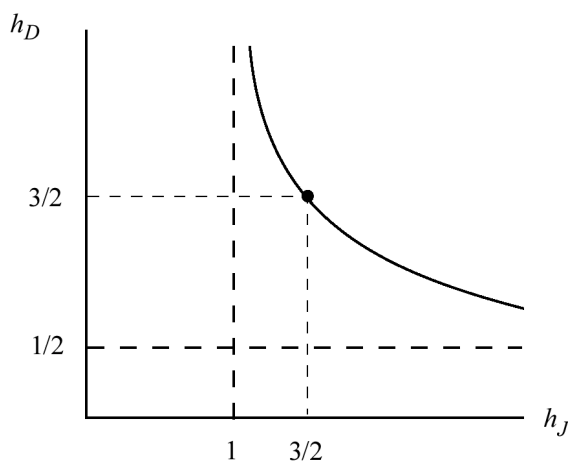
$$\frac{1}{2h_D} + \frac{1}{h_J} = 1$$

or equivalently:

$$h_D = \frac{1}{2}\left(\frac{h_J}{h_J - 1}\right) \tag{1}$$

b) An allocation is Pareto optimal if it satisfies the Samuelson condition and the production function. The production function determines c for any given pair (h_D, h_J), so the Samuelson condition is the only effective restriction on the pair (h_D, h_J). The graph of (1) is shown below. (Note that the graph of (1) actually has two branches. On one of the branches, h_J is smaller than one and h_D is negative. Since Dick can't perform a negative number of hours of housework, the pairs on this branch do not constitute economically sensible solutions. No further consideration will be given to this branch of the graph.)

One of the solutions is the pair $(3/2, 3/2)$, so one of the Pareto optimal allocations involves Dick and Jane each doing 3/2 hours of housework, producing cleanliness of 3. All of the pairs on one side of $(3/2, 3/2)$ require Jane to do more work than Dick, and all of the pairs on the other side require Dick to do more work than Jane.



c) A Nash equilibrium is a pair (h_D, h_J) such that Dick cannot raise his utility by unilaterally changing his hours of housework, and Jane cannot raise her utility by unilaterally changing her hours. This situation can only arise if each person's hours maximizes his or her utility given the other person's hours. Dick's utility is:

$$U_D = (h_D + h_J) - (h_D)^2$$

Dick's hours maximize his utility when:

$$\frac{\partial U_D}{\partial h_D} = 1 - 2h_D = 0 \tag{2}$$

Re-arranging this condition shows that his utility is maximized when he does 1/2 hour of housework, no matter how many hours Jane does. Similarly, Jane's utility is maximized when:

$$\frac{\partial U_J}{\partial h_J} = 1 - h_J = 0 \tag{3}$$

indicating that, no matter how many hours of housework Dick does, she will do 1 hour.

To show that they would both be better off if they each did a little more work, imagine that the Nash equilibrium initially prevails, so that (2) and (3) are satisfied. Now imagine that each of them does a small amount of extra work. Condition (2) states that the extra work that Dick does has no effect on his utility, because the harm caused by the extra work is such offset by the benefit of the extra cleanliness. However, the extra work that Jane does increases cleanliness, making Dick better off. Since his own actions have no effect on his welfare and Jane's actions increase his welfare, Dick is better off when they both chose to work a little more. Similarly, (3) states that Jane's extra work has no effect on her own utility; but since Dick's extra work raises cleanliness, she is better off when they both do a little more work.

Question 2

a) Using Petunia's budget constraint to eliminate c_P from her utility function yields:

$$U_P = 3s^{1/3} + y_P - s$$

Her utility is maximized when no small adjustment in the size of her gardens alters her utility:

$$\frac{dU_P}{ds} = s^{-2/3} - 1 = 0$$

The optimal size of her garden is therefore 1. Her utility is $y_P + 2$. b) Since Joe spends all of his income on private consumption goods and Petunia plants a garden of size 1, Joe's utility is $y_J + 3$.

c) The Shibata box is shown below. Every point in the box corresponds to a commodity bundle (s, c_P).

Initially, Petunia must give up one unit of private consumption goods to increase the size of her garden by one unit. Her budget constraint is therefore a straight line with endpoints $(y_P, 0)$ and $(0, y_P)$. Petunia maximizes her own utility by choosing the commodity bundle A. This bundle lies on her indifference curve U_0 and on Joe's indifference curve V_0.

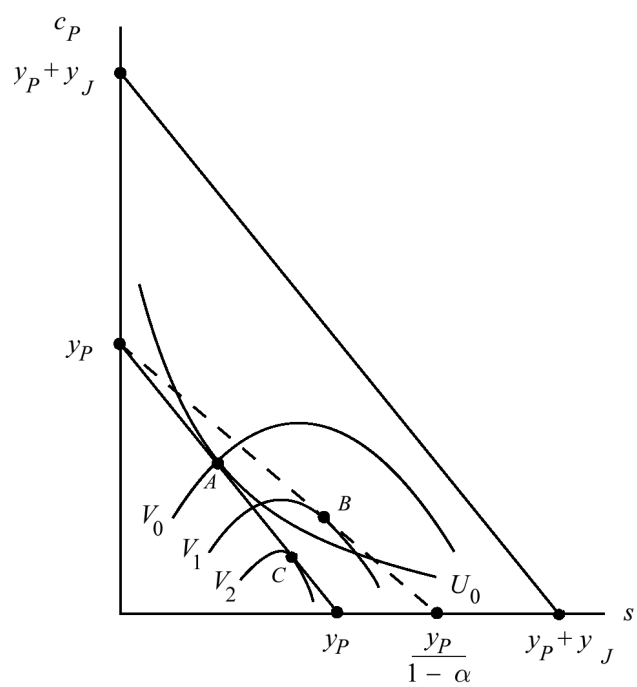
The sharing rule reduces the amount of private consumption goods that she must give up to increase the size of her garden by one unit. The total cost is still one unit of private consumption goods, but since Joe pays α units, Petunia must only pay $1 - \alpha$ units. The reduction in the cost of a garden shifts her budget line outwards, rotating it around its vertical intercept. The intercepts of her new budget line are $(y_P/(1-\alpha), 0)$ and $(0, y_P)$. The budget line for an arbitrarily selected value of α is shown in the figure below.

Petunia tries to persuade Joe to accept this sharing arrangement by allowing him to choose the size of the garden, which effectively means that Joe picks Petunia's commodity bundle.

The feasible commodity bundles lie on the new budget line, and Joe will pick the one that maximizes *his own* utility. This commodity bundle is B, at which one of Joe's indifference curves is just tangent to Petunia's new budget line.

If Joe is offered the sharing rule depicted in the figure, he should certainly accept it, because it allows him to raise his utility from V_0 to V_1. However, not all sharing rules will allow Joe to raise (or even maintain) his utility. The lower the value of α, the flatter is Petunia's budget line. If Petunia's budget line is sufficiently flat—if α is sufficiently low—it will not cut through Joe's indifference curve V_0. That is, every commodity bundle on the budget line would leave Joe worse off than he was initially.

The sharing rule is not without its perils for Petunia, too. Let's look at an extreme case. Imagine that Petunia foolishly sets α at 0. Her "new" budget line would then be the same as the old one, and Joe would pick the commodity bundle C. Since C lies below her initial indifference curve U_0, Petunia would be worse off with this commodity bundle than with her initial bundle A. Furthermore, if Joe's indifference map is "well behaved," the commodity bundle that Joe would pick under values of α near zero would be close to C, and would also lie below U_0. Thus, if α is sufficiently small, the deal makes Harriet worse off.



d) Under the deal offered by Petunia, Joe can pick s to maximize his own utility, subject to the budget constraint:

$$c_J = y_J - \alpha s$$

Substituting this budget constraint into Joe's utility function gives:

$$U_J = 3s^{1/3} + y_J - \alpha s$$

He would maximize his own utility by choosing the garden size that satisfies the condition:

$$\frac{dU_J}{ds} = s^{-2/3} - \alpha = 0$$

implying:

$$s = \alpha^{3/2}$$

Substituting this value of s into Joe's utility function gives Joe's utility under this scheme:

$$U_J = 2\alpha^{-1/2} + y_J$$

But it was argued above that Joe's utility is initially equal to $y_J + 3$, so he is indifferent between accepting and rejecting Petunia's scheme if:

$$2\alpha^{-1/2} = 3$$

implying:

$$\overline{\alpha} = 4/9$$

Chapter 14: Monopoly

Question 1

a) The monopolist is maximizing its profits when it sets output so that marginal cost is equal to marginal revenue. Marginal cost is the increase in total cost associated with an increase in output:

$$MC = \frac{dc}{dq} = 12$$

while marginal revenue is the increase in revenue associated with an increase in output:

$$R = pq = (20 - q)q$$

$$MR = \frac{dR}{dq} = 20 - 2q$$

Equating marginal revenue and marginal cost shows that the firm's profit-maximizing output is 4, and substituting this output into the demand equation shows that the highest price at which the monopolist can sell 4 units of output is 16.

b) The government should require the monopolist to operate at the highest level of output at which its revenues cover its costs—that is, the highest level of output at which price is at least as high as average cost. This level of output is characterized by the condition:

$$AC = p$$

Here, AC is average cost:

$$AC = \frac{c}{q} = \frac{15}{q} + 12$$

and the market price p is determined by the demand curve. Substituting into the above condition gives:

$$\frac{15}{q} + 12 = 20 - q$$

or:

$$q^2 - 8q + 15 = 0$$

This quadratic equation has two roots:

$$(q-3)(q-5) = 0$$

Since the monopolist should be forced to produce the highest level of output at which revenues cover costs, the required level of output is 5. The highest price at which this quantity of goods can be sold is 15.

c) The firm should expand its output until the value of one more unit of goods to the consumers is just equal to the cost of producing one more unit:

$$p = MC$$

Substituting into this condition shows that output should be 8 units; the price at which these units can be sold is 12. The firm's losses are the difference between its costs and its revenue:

$$c - R = 111 - 96 = 15$$

Question 2

Firm i chooses s_i to maximize its own profits, implying:

$$\frac{d\pi_1}{ds_1} = \left(\frac{1}{s_1 + s_2}\right)^2 s_2 V_1 - 1 = 0$$

$$\frac{d\pi_2}{ds_2} = \left(\frac{1}{s_1 + s_2}\right)^2 s_1 V_2 - 1 = 0$$

Define z to be the total resources expended by the two firms. Then the Nash equilibrium can be described by the three equation system consisting of these two conditions and the definition of z:

$$z^2 = s_2 V_1$$

$$z^2 = s_1 V_2$$

$$z = s_1 + s_2$$

Eliminate s_1 and s_2 by substituting the first two equations into the third:

$$\frac{z}{V_2} + \frac{z}{V_1} = 1$$

$$z = \frac{V_1 V_2}{V_1 + V_2}$$

Substitute this value back into the first two equations (of the three equation system) to obtain the Nash equilibrium values of s_1 and s_2:

$$s_1 = \frac{(V_1)^2 V_2}{(V_1 + V_2)^2}$$

$$s_2 = \frac{(V_2)^2 V_1}{(V_1 + V_2)^2}$$

Now evaluate the probabilities:

$$P_i = \frac{V_i}{V_1 + V_2}$$

Finally, substituting the resource expenditures and the probabilities into the expected profit equation shows that:

$$\pi_i = \frac{(V_i)^3}{(V_1 + V_2)^2} > 0$$

Chapter 15: Pricing Rules under Imperfect Competition

Question 1

The highest price at which q units of goods can be sold is:

$$p = 5 - \frac{q}{200}$$

Firm 1's profits are the difference between its revenues and its cost:

$$\pi_1 = \left(5 - \frac{q_1 + q_2}{200}\right) q_1 - 2q_1$$

and its profit-maximizing output satisfies the condition:

$$\frac{\partial \pi_1}{\partial q_1} = 3 - \frac{2q_1 + q_2}{200} = 0$$

Re-arranging this equation gives firm 1's best response function:

$$q_1 = 300 - (q_2/2)$$

Since the firms are symmetrical, firm 2's best response function is:

$$q_2 = 300 - (q_1/2)$$

The Cournot equilibrium is a pair (q_1, q_2) that satisfies both best response functions. It is readily established that it is $(200, 200)$. The highest price at which 400 units can be sold is three dollars, which exceeds marginal cost.

Question 2 The Bertrand equilibrium is $(2, 2)$. Each firm sells 300 units of goods, but neither firm earns a profit because revenues are just equal to costs. To see why this pair is a Bertrand equilibrium, consider the possible unilateral deviations. If a firm were to raise its price, it would lose all of its customers. This deviation would not reduce the firm's profits (its profits would still be zero) but it wouldn't raise them. If a firm were to reduce its price, it would attract all of the buyers, but it would be selling each unit of goods for less than its cost of production, and therefore would incur a loss. Again, this deviation does not raise profits. Since there are no profitable deviations, $(2, 2)$ is a Bertrand equilibrium. Let's check other configurations to ensure that there are no other Bertrand equilibria:

- If the prices are above marginal cost and are not equal, the high price firm has no sales and no profits. It could earn positive profits by slightly undercutting the other firm's price. If the prices are above marginal cost and equal, each firm is earning positive profits. Each firm could double its sales and its profits by slightly undercutting the other firm's price. The existence of a profitable deviation implies that none of these configurations is a Bertrand equilibrium.
- If one of the prices is equal to marginal cost and the other price is above marginal cost, the low-price firm has all the customers but no profits. It could earn positive profits by slightly raising its price. Again, the existence of a profitable deviation implies that none of these configurations is a Bertrand equilibrium.
- Finally, if one or both of the prices is below marginal cost, at least one of the firms is selling goods at a loss. It could increase its profits (to zero) by raising its price above that of the other firm. Consequently, these configurations cannot be Bertrand equilibria.

Question 3 The Bertrand equilibrium is $(1.99, 2.00)$. Again, this pair is shown to be a Bertrand equilibrium by showing that there are no profitable deviations for either firm:

- Firm 2 has no customers and no profits, but if it lowered its price to gain customers, it would be selling goods for less than their cost of production. It would continue to have no customers and no profits if it raised its price.
- Firm 1 has all of the customers and positive profits. If it raised its price, it would lose some or all of its customers, and its profits would be lower. Suppose instead that it reduced its price. It would have more customers (because the demand curve is downward sloping) but earn lower profits on each unit sold. Specifically, its profits would be:

$$\pi_1 = (1000 - 200p)(p - 1) \qquad p < 2$$

so:

$$\frac{d\pi_1}{dp} = 1200 - 400p$$

This derivative is positive at every p less than 2, so any price reduction would reduce firm 2's profits.

The absence of any profitable deviations implies that $(1.99, 2.00)$ is a Bertrand equilibrium. There is no other Bertrand equilibrium.

What is the role of the assumption that the price must be measured in dollars and cents? Without it, there would be no equilibrium. Firm 1 would want to undercut firm 2, so it would set its price below 2. However, whatever price smaller than 2 it chose, it could increase its profits slightly by slightly increasing its price. If its price were 1.99, it could increase its profits by raising its price to 1.999, and if its price were 1.999, it could increase its profits by raising its price to 1.9999. Since firm 1 would have a profitable deviation, no matter how close its price was to 2, there would be no equilibrium.

Note that imposing the same assumption in question 2 would generate a second equilibrium: $(2.01, 2.01)$. Here, each firm would set its price above marginal cost by the smallest possible amount, and each firm would be earning small profits.

Chapter 16: Taxation

Question 1

a) Robinson's budget constraint is:

$$c = 20h$$

Substituting this constraint into his utility function gives:

$$U = 20h - h^2$$

His utility is maximized when:

$$\frac{dU}{dh} = 20 - 2h = 0$$

Robinson collects coconuts for 10 hours and consumes 200 coconuts. His utility is 100.

b) The coconuts that Robinson collects are either consumed or confiscated by the empire, so Robinson's budget constraint is:

$$c + 32 = 20h$$

Substituting this constraint into his utility function gives:

$$U = 20h - 32 - h^2$$

Proceeding as before, it can be shown that Robinson continues to work 10 hours each month and collect 200 coconuts. He only gets to consume 168 of them, however, so his utility is 68.

c) Under the income tax, Robinson consumes four-fifths of the coconuts collected:

$$c = (4/5)(20h)$$

Substituting this constraint into Robinson's utility function gives:

$$U = 16h - h^2$$

His best attainable commodity bundle satisfies the condition:

$$\frac{du}{dh} = 16 - 2h = 0$$

so Robinson works 8 hours, collects 160 coconuts. Robinson consumes four-fifths of the coconuts (128) and gives one-fifth (32) to the empire.

Under the non-contingent tax, Robinson pays 32 coconuts in taxes and has a utility of 68. Under the income tax, Robinson pays 32 coconuts in taxes and has a utility of 64. This fall in welfare arises from Robinson's attempts to avoid paying taxes by reducing his hours of work. The decline in his utility (from 68 to 64) could be reversed by giving Robinson 4 coconuts, so the welfare cost of the income tax, measured in coconuts, is 4.

Question 2

a) Here is a general solution to Andy's utility maximization problem. If his after-tax income is y and the tax-inclusive prices of ale and bread are p_a and p_b respectively, his budget constraint is:

$$p_a a + p_b b = y \tag{1}$$

He can choose any commodity bundle (a, b) that satisfies this constraint. The best of these bundles also satisfies the condition that the rate at which he is just willing to trade ale for bread is equal to rate at which the market allows him to do so:

$$MRS = \frac{p_a}{p_b}$$

Evaluating the marginal rate of substitution in the usual way gives:

$$\frac{b}{2a} = \frac{p_a}{p_b} \tag{2}$$

Andy's best attainable commodity bundle satisfies (1) and (2). Since (1) states that Andy spends all of his income on ale and bread, and (2) states he spends twice as much on bread as he does on ale, Andy spends one-third of his income on ale and two-thirds of his income on bread:

$$a^\circ = \left(\frac{1}{3}\right)\left(\frac{y}{p_a}\right) \tag{3}$$

$$b^\circ = \left(\frac{2}{3}\right)\left(\frac{y}{p_b}\right) \tag{4}$$

In the case at hand, y is \$27 and p_a and p_b are both \$1, so Andy buys 9 pints of ale and 18 loaves of bread.

b) Andy loses \$3 to the government, so his after-tax income is \$24. His budget constraint is:

$$a + b = 24$$

Substituting into (3) and (4) shows that he consumes 8 pints of ale and 16 loaves of bread.

c) Andy's income is again \$27, but the price of each commodity has risen from \$1 to \$(9/8). Substituting into (3) and (4) shows that he again consumes 8 pints of ale and 16 loaves of bread. He pays the government a tax of \$(1/8) on each of the 24 units of goods that he buys, so his total tax payment is \$3. That is, raising both prices by $12^1/_2\%$ has the same impact as a lump-sum tax of \$3.

One way to explain their equivalence is to note that Andy's budget constraint under the proportional tax is:

$$(9/8)a + (9/8)b = 27$$

or:

$$a + b = 24$$

Thus, the proportional tax leads to the same budget constraint as the lump-sum tax. Since he is confronted with the same group of attainable commodity bundles under each tax, he will make the same choice under each tax.

d) Andy's income is \$27, and p_a and p_b are \$1 and \$(6/5) respectively. He consumes 9 pints of ale and 15 loaves of bread. Since he pays \$(1/5) on each loaf of bread, his tax payment is \$3—as it was under each of the previous taxes.

e) Andy's income is \$27, and p_a and p_b are \$(3/2) and \$1 respectively. He consumes 6 pints of ale and 18 loaves of bread. He pays \$(1/2) in taxes on each pint of ale, so his tax payment is again \$3.

f) Numerically, Andy's utilities under the proportional tax, the bread tax and the ale tax are 12.70, 12.65 and 12.48 respectively. The last two taxes generate a deadweight loss while the first tax does not.

Chapter 17: The Welfare Cost of Tax Interactions

a) The Marshallian demands can be found as the solution to a two equation system. One equation is the budget constraint, and the other is the condition for a tangency between the budget constraint and an indifference curve. However, you probably know by now that an individual with a Cobb-Douglas utility function spends a fixed fraction of his income on each good (with the sum of the fractions being one), and that the fraction spent on each good is proportional to the exponent attached to that good in the utility function. Applying this rule to the present case gives:

$$a^\circ = \frac{1}{3}\frac{Y}{p_a}$$

$$b^\circ = \frac{2}{3}\frac{Y}{p_b}$$

b) The production possibility frontier is:

$$2a + b = 150$$

The right-hand side of this equation is the amount of labour available, and the left-hand side is the amount of labour needed to produce any commodity bundle (a, b).

John's labour income is 150 dollars, and with no taxes in place, p_a is 2 dollars and p_b is 1 dollar. Substituting these parameters into the Marshallian demand equations shows that John consumes 25 pints of ale and 100 loaves of bread. This commodity bundle requires 150 hours of labour to produce, and hence lies on the production possibility frontier.

c) John consumes all of the goods produced in the economy when his best attainable commodity bundle (a°, b°) lies on the frontier:

$$2a^\circ + b^\circ = 150$$

Substituting the Marshallian demands into this equation gives the income that John must have to purchase all of the economy's output:

$$\frac{2Y}{3}\left(\frac{1}{p_a} + \frac{1}{p_b}\right) = 150$$

$$Y = 225 \div \left(\frac{1}{p_a} + \frac{1}{p_b}\right) \tag{1}$$

The government's tax revenue is:

$$R = (p_a - 2)a^\circ + (p_b - 1)b^\circ$$

Substituting John's income (1) into the Marshallian demands, and the Marshallian demands into the revenue equation gives:

$$R = \frac{225p_ap_b - 150p_b - 150p_a}{p_a + p_b} = \frac{225p_ap_b}{p_a + p_b} - 150 = Y - 150$$

That is, when John's income is given by (1), the government earns just enough revenue to provide him with the transfer needed to raise his income to that level.

d) Equation (1) gives John's income when the tangency between his budget constraint and one of his indifference curves occurs at a point on the production possibility frontier. That is, it is John's income in any competitive equilibrium. Substituting (1) into the Marshallian demand equations gives the GE demand curves. When p_b is equal to one,

$$a_{GE} = \frac{75}{1 + p_a}$$

and when p_a is equal to two,

$$b_{GE} = \frac{300}{p_b + 2}$$

Chapter 18: The Theory of the Second Best

a) The budget constraint is:

$$c = (1-t)h$$

Substituting the budget constraint into the utility function gives:

$$U = 2\left((1-t)^{1/2}h^{1/2} + z^{1/2}\right) - h$$

The utility-maximizing hours of work satisfy the condition:

$$\frac{dU}{dt} = (1-t)^{1/2}h^{-1/2} - 1 = 0$$

implying:

$$h = 1 - t$$

Substituting these hours of work into the utility function gives:

$$U = (1-t) + 2z^{1/2} \tag{1}$$

b) Since the government finances the public good through the income tax,

$$900z = 400t(1-t)$$

(Note that each person's taxes are $t(1-t)$ because each person's income is $1-t$.) Re-arranging this condition gives:

$$z = (4/9)\,t(1-t) \tag{2}$$

Using this equation to eliminate z from (1) gives:

$$U = (1-t) + (4/3)\left[t(1-t)\right]^{1/2}$$

c) The government sets the tax rate to maximize the typical person's utility:

$$\frac{dU}{dt} = -1 + (2/3)\left[t(1-t)\right]^{-1/2}(1-2t) = 0 \tag{3}$$

Substitution shows that this condition is satisfied when t is 0.2.

d) Each unit of public good costs \$900, so one dollar buys 1/900 unit of public good. The marginal utility of each unit of public good is $z^{-1/2}$, so spending one more dollar increases utility by:

$$(1/900)z^{-1/2}$$

Since the public good is financed through the income tax, (2) is satisfied. The increase in utility is therefore:

$$(1/600)\left[t(1-t)\right]^{-1/2}$$

e) Differentiating (1) gives:

$$\frac{dU}{dt} = -1$$

so utility falls by one when the tax rate is marginally increased. Since each person's income tax is $t(1-t)$, the increase in total tax revenue when the tax rate is marginally raised is:

$$\frac{dR}{dt} = \frac{d}{dt}\left[400t(1-t)\right] = 400(1-2t)$$

The loss of utility when one additional dollar of revenue is raised, $dU/dt \div dR/dt$, is:

$$\frac{1}{400(1-2t)}$$

f) Equating the two changes in utility gives:

$$(1/600)\left[t(1-t)\right]^{-1/2} = \frac{1}{400(1-2t)}$$

or:

$$(2/3)\left[t(1-t)\right]^{-1/2}(1-2t) = 1$$

which is equivalent to (3).

Chapter 19: Asymmetric Information

Question 1 a) If θ is uniformly distributed on the interval $[0, \overline{\theta}]$, the average value of θ is $\overline{\theta}/2$. Here, $\overline{\theta}$ is 1, so the average value of θ is 1/2 and the average daily output of the workers is 3/2.

b) Since only the best workers leave, abilities after the buy-out are uniformly distributed on the interval [0,4/5]. Average daily output is therefore $1 + 2/5$, or 7/5.

c) The random firing of workers does not change the distribution of abilities, since workers of different abilities are equally likely to be fired. The average daily output of the workers will still be 3/2.

d) After the buy-out, the firm's workforce consists of two groups. The first group consists of the workers who did not ask for the buy-out. Their abilities are uniformly distributed on the interval $[0, 3/5]$, so their average daily output is 13/10 (*i.e.*, $1 + 3/10$). The second group consists of the workers who wanted the buy-out but were refused. The abilities of the workers who *wanted* the buy-out are uniformly distributed on the interval $[3/5, 1]$, so their average output was 9/5 (*i.e.*, $1 + 4/5$). (Note that you can think of these workers as having abilities that satisfy the condition:

$$\theta = 3/5 + \gamma$$

where γ is uniformly distributed on the interval $[0, 2/5]$. The average value of θ for this group is therefore 4/5.) Since the workers who actually received the buy-out were randomly drawn from this group, the abilities of the group who received the buy-out and of the group who

were refused the buy-out are uniformly distributed on the same interval. Thus, the average daily output of workers in the second group is 9/5.

After the buy-out, 3/4 of the workforce consists of people who did not request the buy-out and 1/4 consists of people who requested the buy-out and were refused it. Average daily output for the workforce is therefore $(3/4)(13/10) + (1/4)(9/5)$, or 57/40.

Question 2

a) Under full insurance, an apple picker will accept insurance if:

$$EU_1 \geq EU_0$$

when z is equal to 1. This condition is satisfied if:

$$(1-p)^{1/2} \geq \theta$$

or equivalently,

$$p \leq 1 - \theta^2 \tag{1}$$

The insurer's expected profits on a single contract are:

$$E\pi = p - (1-\theta) \tag{2}$$

because he receives p with certainty and spends 1 with probability $1-\theta$. This expression is positive when p is equal to $1-\theta^2$ which, by (1), is the highest premium that a worker will pay.

b) Substituting into (1) shows that awkward workers will accept insurance if p is not greater than 5/9, and that clumsy workers will accept insurance if p is not greater than 8/9. Thus, the highest premium at which both kinds of apple pickers will accept insurance is 5/9. Substituting into (2) shows that, if this premium is charged,

$$E\pi_a = 2/9$$

$$E\pi_c = -1/9$$

Expected profits on any given contract are:

$$E\pi = (2/9)\alpha - (1/9)(1-\alpha)$$

so that expected profits are positive if α is greater than 1/3, negative if it is less than 1/3, and equal to zero if it is equal to 1/3.

c) When z is equal to 1/4, an apple picker will accept insurance if:

$$EU_1 \geq EU_0$$

when z is equal to $1/4$. Then:

$$(1-p)^{1/2}\left[\theta + (1/2)(1-\theta)\right] \geq \theta$$

$$(1-p)^{1/2} \geq \frac{2\theta}{1+\theta}$$

$$p \leq 1 - \left(\frac{2\theta}{1+\theta}\right)^2$$

Thus, an awkward apple picker will accept insurance if p is less than 9/25 and a clumsy apple picker will accept insurance if p is less than 3/4.

When z is equal to 1/4, the expected profits on a contract with awkward and clumsy apple pickers are:

$$E\pi_a = (2/3)p - (1/3)(1-p)(1/4) = (3/4)p - (1/12)$$

$$E\pi_c = (1/3)p - (2/3)(1-p)(1/4) = (1/2)p - (1/6)$$

(If an apple picker does not fall out of the tree, the insurer receives the fraction p of the apple picker's income as a premium. If the apple picker does fall out of a tree, the insurer pays him an income of 1/4, but a fraction p of that income is returned to the insurer as a premium.) The highest premium at which both kinds of apple pickers will buy insurance is 9/25, and at this premium,

$$E\pi_a = 14/75$$

$$E\pi_c = 1/75$$

Since both of these numbers are positive, the insurers expected profits will be positive under any α. That is, it can be profitable for insurers to provide partial insurance even if it is unprofitable to provide full insurance.

Question 3

a) Simple calculations show that a cane cutter's expected utility is 27/16 if he is careful and 2/3 if he is not. He is better off if he is careful.

b) The expected utility of a careful insured cane cutter is:

$$EU_1 = 9\,(1-p)^{1/2} - \frac{7}{16}(6) - \frac{3}{4}$$

while the expected utility of a careless insured cane cutter is:

$$EU_0 = 9\,(1-p)^{1/2} - \frac{5}{9}(6)$$

The latter expected utility is higher than the former for every value of p, so insured cane cutters will not take precautions.

c) Since insured cane cutters will be careless, an insurer's expected profits on each contract are:

$$E\pi = p - \frac{5}{9}$$

Competition will drive the premium to $5/9$. The expected utility of a careless insured cane cutter who must pay this premium is 8/3, which is higher than the expected utility of an uninsured cane cutter (27/16). The cane cutters will therefore accept the insurance.

d) If every cane cutter were careful, an insurer's expected profits on each contract would be:

$$E\pi = p - \frac{7}{16}$$

so that the insurer's expected profits would be zero when the premium is 7/16. The expected utility of a cane cutter would then be 27/8, which is greater than the utility that the cane cutters actually achieve under insurance (8/3). Everyone would be better off if everyone would take precautions—but no one does.

Question 4 Let A be the expected profits of the owner, and let B be the expected profits of the agent. Then:

$$A = (1/2)\{-2x\} + (1/2)\{8xy + (1-y)R\}$$

$$B = (1/2)\{0\} + (1/2)\{y[10(1-x) - 50x] + (1-y)[10-R]\}$$

Consider A first:

- If the *actual* sale price is zero, the agent will truthfully report this sale price. If the owner accepts the report, he earns profits of zero. If he rejects the report (which he does with probability x), he pays two dollars but still does not collect anything from the agent. Thus, if the actual sale price is zero, his expected profits are $-2x$.
- If the *actual* sale price is ten dollars, the agent truthfully reports the sale price with probability $1-y$, allowing the owner a profit of R. However, with probability y, the agent will claim that the sale price was zero. The owner then only earns a profit if he rejects the report (which he does with probability x), and then his profits are only eight dollars (ten dollars less the two dollar cost of a rejection). Thus, if the actual sale price is ten dollars, his expected profits are $8xy + (1-y)R$.
- Since the actual sale price takes each of these values with probability 1/2, the owner's expected profits are A.

Now consider B:

- If the actual sale price is zero, the agent gets nothing.
- If the actual sale price is ten dollars, the agent truthfully reports the sale price with probability $1-y$ and lies with probability y. If he is truthful, he earns $10-R$. If he lies, his report is accepted with probability $1-x$, allowing him to keep the entire ten dollars, and his report is rejected with probability x, causing him to lose fifty dollars. Consequently, his expected profits are $y[10(1-x) - 50x] + (1-y)[10-R]$.
- Each of these outcomes occurs with probability 1/2, so the agent's expected profits are B.

To find the Nash equilibrium, first find each person's best response function. The owner's expected profits can be written as:

$$A = (1/2)[x(8y-2) + (1-y)R]$$

His expected profits are maximized by following the rule:

If y is greater than 1/4, always reject a report that the object was worthless. If y is less than 1/4, never reject a report that the object was worthless. If y is equal to 1/4, choose any probability of rejection.

or equivalently,

$$x^* = \begin{cases} 1 & \text{if } y > 1/4 \\ [0,1] & \text{if } y = 1/4 \\ 0 & \text{if } y < 1/4 \end{cases}$$

The agent's expected profits can be written as:

$$B = (1/2)\left[y(R - 60x) + 10 - R\right]$$

so his expected profits are maximized by following the rule:

If x is less than $R/60$, always lie. If x is greater than $R/60$, never lie. Otherwise, choose any probability of lying.

Mathematically, this rule is:

$$y^* = \begin{cases} 1 & \text{if } x < R/60 \\ [0,1] & \text{if } x = R/60 \\ 0 & \text{if } x > R/60 \end{cases}$$

Inspection of these rules quickly reveals that there is no Nash equilibrium in which one or both of the players do not behave randomly. (For example, if the owner never questions a bad report, the agent should always cheat; but if the agent always cheats, the owner should always question a bad report.) The only Nash equilibrium is $(R/60, 1/4)$.

Substituting the equilibrium values of x and y into A and B gives:

$$A = 3R/8$$

$$B = (1/2)(10 - R)$$

Clearly, the owner makes himself as well off as possible by setting R at its highest permissible value (10). When he does so,

$$A = 3^3/_4$$

$$B = 0$$

so that:

$$A + B = 3^3/_4$$

But the expected value of the object itself is 5, so the strategic behaviour engendered by the owner's inability to observe the sale price results in an expected loss of $1^1/_4$ to this partnership.

Chapter 20: Preference Revelation

Question 1

Each person has the same marginal rate of substitution:

$$MRS_i = \frac{1}{2}\left(\frac{c_i}{z}\right)$$

Substituting the marginal rates of substitution and the marginal rate of transformation into the Samuelson condition gives:

$$\frac{1}{2}\left(\frac{c_A + c_B + c_C}{z}\right) = 20$$

For each person, consumption is determined by the equation:

$$c_i = y_i - t_i z \tag{1}$$

where, under the Lindahl equilibrium, the tax price is equal to the marginal rate of substitution:

$$t_i = \frac{1}{2}\left(\frac{c_i}{z}\right) \tag{2}$$

Equations (1) and (2) can be solved for person i's private consumption and tax price:

$$c_i = \frac{2}{3}(y_i)$$

$$t_i = \frac{1}{3}\left(\frac{y_i}{z}\right)$$

Substituting the private consumptions into the Samuelson condition gives:

$$\frac{y_A + y_B + y_C}{z} = 60$$

$$z = 6$$

The private consumptions and tax prices can now be evaluated:

$$c_A = 60,\ c_B = 80,\ c_C = 100$$

$$t_A = 5,\ t_B = 20/3,\ t_C = 25/3$$

Question 2

a) The economy is in a Nash equilibrium if no-one has an incentive to alter their claims of their types. Since type H and type L people have no incentive to lie, only the behaviour of type M people need be examined. Their ideal level of public goods provision is:

$$\widehat{z}_M = (120)^{1/(1-\alpha)}$$

The amount of public good provided by the government will be:

$$z^{**} = \left(10\widetilde{h} + 4\widetilde{m} + 2(30 - \widetilde{h} - \widetilde{m})\right)^{1/(1-\alpha)}$$

where $\widetilde{h}$ and $\widetilde{m}$ are the numbers of people who claim to be type H and type M respectively. A sufficient condition for a Nash equilibrium is that the reports of the type M people cause the government to produce exactly the quantity of public good that the type M people want. This outcome occurs if:

$$10\widetilde{h} + 4\widetilde{m} + 2(30 - \widetilde{h} - \widetilde{m}) = 120 \tag{3}$$

(If the type M people can't get exactly what they want, there would be a Nash equilibrium in which they get *close* to what they want. We would have to look for this kind of equilibrium if we couldn't find an equilibrium that satisfies the above condition.) If everyone were to tell the truth, the value of the expression on the left-hand side of (3) would be 130. However, every type M who claims to be type L would lower the value of this expression by 2. If 5 of the type M people claim to be type L, z^{**} would match $\widehat{z}_M$, and none of the type M people would have an incentive to change their behaviour. Thus, the Nash equilibrium has all of the type H and type L people telling the truth, and 10 of the type M people telling the truth, and 5 of the type M people claiming to be type L.

b) Again, there is a Nash equilibrium in which people arrange themselves so that (3) is satisfied. If everyone tells the truth, the value on the left-hand side of this expression would be 90, and type M people would have an incentive to misrepresent their preferences to raise this value. Every type M person who claims to be type H would raise the value of this expression by 6. If 5 of them did so, the value of this expression would be 120—which is exactly what type M people want. Thus, the Nash equilibrium has all of the type L people telling the truth, and 10 of the type M people telling the truth, and 5 of the type M people claiming to be type H.

Chapter 21: Regulation of a Natural Monopoly

a) Since raising revenue is costly, the government should set M as low as possible—that is, M should be equal to θe^2. Under this rule, the net social value when θ is low is:

$$W = 12e - 3e^2$$

The best value of e satisfies the condition:

$$12 - 6e = 0$$

implying that e_L is 2. The payment that just compensates the firm for its costs, M_L, is 8. If θ is high, the net social value is:

$$W = 12e - 6e^2$$

The best value of e satisfies the condition:

$$12 - 12e = 0$$

implying that e_H is 1. The payment that just compensates the firm for its costs, M_H, is 4.

b) The government offers the contract:

Work either 2 weeks or 1 week. If you work for 2 weeks, you will be paid 8 thousand dollars. If you work for 1 week, you will be paid 4 thousand dollars.

Suppose that the actual value of θ is 2. The firm would earn zero profits if it worked for two weeks. However, if it worked for only one week, its profits would be $4 - 2(1)^2$, or 2. Since it prefers positive profits to no profits, the firm will work on the project for only one week, and not for two weeks. Consequently, W is not maximized when θ is 2.

c) These constraints are, in order:

$$M_L - 2\left(e_L\right)^2 \geq 0$$

$$M_H - 4\left(e_H\right)^2 \geq 0$$

$$M_L - 2\left(e_L\right)^2 \geq M_H - 2\left(e_H\right)^2$$

$$M_H - 4\left(e_H\right)^2 \geq M_L - 4\left(e_L\right)^2$$

d) The binding constraints can be written as:

$$M_H - 4\left(e_H\right)^2 = 0$$

$$M_L - 2\left(e_L\right)^2 = M_H - 2\left(e_H\right)^2$$

Using these equations to eliminate M_L and M_H from the expression for the expected net social value of the public good yields:

$$EW = (1/2)\left[12e_L - 3(e_L)^2\right] + (1/2)\left[12e_H - 7(e_H)^2\right]$$

The characteristic of the optimal pair (e_L, e_H) is that a small variation in either value leaves the expected net social value of the public good unchanged:

$$\frac{\partial EW}{\partial e_L} = 12 - 6e_L = 0$$

$$\frac{\partial EW}{\partial e_H} = 12 - 14e_H = 0$$

Thus, the optimal values of e_L and e_H are 2 and 6/7 respectively. (Note that e_H is smaller under asymmetric information than under full information.) The binding constraints can then

be used to determine the payments:

$$M_H = 4\left(\frac{6}{7}\right)^2$$

$$M_L = 8 + 2\left(\frac{6}{7}\right)^2$$

(Note that M_L is larger under asymmetric information than under full information, even though e_L is the same under both information structures.)

Chapter 22: Other Examples of Asymmetric Information

Question 1 a) The agent will report the sale price that minimizes the amount that he must pay to the owner. One way of doing so is to report that the sale price is zero, since under this report, his payment must be zero. Thus, whatever the actual sale price, the payment received by the owner will be zero.

b) The agent will tell the truth if there is no benefit to lying. To find the schedule under which he always tells the truth, first note that if he were to lie, the best lie would be to report that the sale price was zero. No payment would then be required of him, and he would be able to keep the entire sale price of the asset if he is not caught. If he is caught, of course, he gets nothing from the sale of the asset and pays a fine P. The expected pay-off from cheating is therefore $(1-\pi)S - \pi P$ when the actual sale price is S. The expected pay-off from not cheating is $S - R$, where R is the payment that must be made when S is reported. The agent will have no incentive to cheat if:

$$S - R \geq (1-\pi)S - \pi P$$

or:

$$R \leq \pi(S+P)$$

An incentive compatible contract must satisfy this condition at every S. The incentive compatible contract that makes the owner as well off as possible is the contract under which R is as big as possible under every S. If $\pi(S+P)$ does not exceed S, the biggest possible R is $\pi(S+P)$. If $\pi(S+P)$ exceeds S, the biggest possible R is S. That is, the best schedule is:

$$R = \min[S, \pi(S+P)]$$

The amount retained by the agent, C, is:

$$C = S - R = \max[0, (1-\pi)S - \pi P]$$

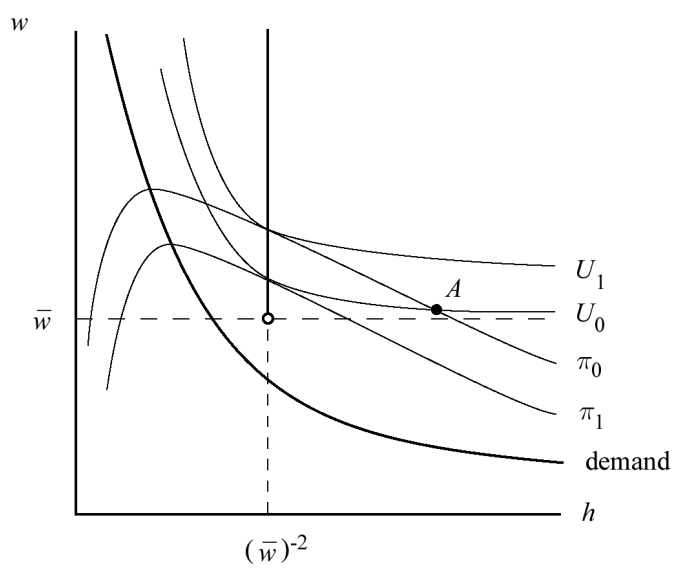
Question 2 a) The increase in efficiency causes the value of labour's marginal product to rise. Since firms hire labour up to the point where the wage is equal to the value of labour's marginal product, the demand for labour curve is shifted upwards. The wage and employment

rise. The effect on the welfare cost is uncertain. There is less unemployed labour, but each unit of unemployed labour has a higher social cost, because more output is forgone when labour is not used.

b) The increased efficiency of the labour market reduces the loss experienced by a worker who loses his job, causing the "no shirking" locus to shift. Which way does it move? At every point on the old locus, e is now greater than pL (because L has fallen). To reach a point on the new locus, $L(w, w, u)$ must be pushed up until $pL(w, w, u)$ is once again equal to e. This increase can occur through either an increase in w or an increase in u, implying that the new locus lies above and to the left of the old locus. The wage rises and employment falls. The welfare cost unambiguously rises because there is greater unemployment.

Chapter 23: The Distribution of Income

Question 1



a) Since the wage cannot be below $\overline{w}$, only wages higher than this value need be considered. So long as the wage is above the minimum wage, the union's utility rises with w and h, so utility can only be held constant if one of these variables rises while the other falls. The usual rule determines its slope:

$$\text{slope of indifference curve} = -\left(\partial U/\partial h \div \partial U/\partial w\right) = \frac{\overline{w} - w}{h}$$

Inspection of this expression shows that the indifference curves are asymptotic to the vertical axis, and to the horizontal line with height $\overline{w}$. b) The value of h that maximizes the firm's

profits satisfies the condition:

$$\frac{\partial \pi}{\partial h} = h^{-1/2} - w = 0$$

so the equation for the demand curve is:

$$h = w^{-2}$$

Clearly, the quantity of labour demanded under competition falls as the wage rises.

c) Consider first the slope. Evaluating the partial derivatives gives:

$$\text{slope of iso-profit curve} = \frac{h^{-1/2} - w}{h}$$

The numerator of this expression is zero on the competitive demand curve, so the iso-profit curve has a slope of zero as it crosses the competitive demand curve. Given any initial point on the demand curve, a point to the left of the demand curve is reached by reducing h or w. Either of these changes would increase the value of the numerator from zero, so the iso-profit curve has a positive slope at such points. Similarly, a point to the right of the demand curve is reached by increasing h or w, so that the numerator falls from zero. It follows that the iso-profit curve is negatively sloped at these points. Now consider the stacking of the iso-profit curves. For any given h, a fall in w increases profits—that is, results in a move to an iso-profit curve associated with a higher level of profits.

d) Let the point A be $(\widehat{h}, \widehat{w})$. The indifference curve U_0 and the iso-profit curve π_0 pass through A, forming the boundary of a lens-shaped area that lies above and to the left of A. Every pair (h, w) in the interior of the lens is on an indifference curve that lies farther from the origin than U_0, and is therefore preferred to $(\widehat{h}, \widehat{w})$ by the union. Every pair (h, w) in the interior of the lens also lies on an iso-profit curve that is lower in the figure—and therefore corresponds to a higher level of profits—than π_0. Thus, both the firm and the union prefer every point in the interior of the lens to $(\widehat{h}, \widehat{w})$.

If the indifference curve and iso-profit curve are tangent to each other, any alternative to $(\widehat{h}, \widehat{w})$ that lies on a higher indifference curve also lies on an iso-profit curve corresponding to lower profits, and any alternative that yields higher profits lies on a lower indifference curve. Thus, any move from $(\widehat{h}, \widehat{w})$ makes the union better off at the firm's expense, or the firm better off at the union's expense.

e) A tangency between an indifference curve and an iso-profit curve can only occur above the competitive demand curve, where the iso-profit curves are downward sloping. At every tangency, the two curves have the same slope:

$$\frac{\overline{w} - w}{h} = \frac{h^{-1/2} - w}{h}$$

or:

$$\overline{w} = h^{-1/2}$$

Thus, every tangency occurs when h is equal to $\overline{w}^{-2}$. Furthermore, the tangency condition is satisfied for every w greater than $\overline{w}$.

Question 2

a) Substitution gives:

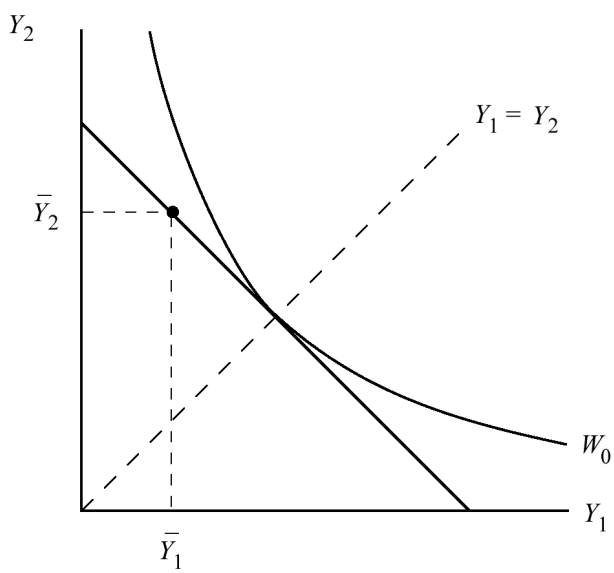
$$W = (Y_1)^\gamma (Y_2)^\gamma$$

where γ is equal to $\alpha\beta$. The slope of the social indifference curves in the (Y_1, Y_2) quadrant is found in the usual way:

$$\text{slope of social indifference curve} = -\left(\frac{\partial W}{\partial Y_1} \div \frac{\partial W}{\partial Y_2}\right) = -\frac{Y_2}{Y_1} \tag{1}$$

The indifference curves have the customary shape and are asymptotic to both axes.

b) The figure below shows the utility possibility frontier: it is a straight line with slope of -1. This frontier shows society's options, and the social indifference curves show the manner in which society ranks options. The best income distribution is that at which one of the social indifference curves is just tangent to the utility possibility frontier, because every other income distribution lies on a lower social indifference curve.



Tangent curves have the same slope, and since the frontier has a slope of -1 everywhere, the social indifference curve must have a slope of -1 at the point of tangency. By (1), the incomes are equal at the point of tangency.

The utility possibility frontier is the same for every pair $(\overline{Y}_1, \overline{Y}_2)$ satisfying the condition:

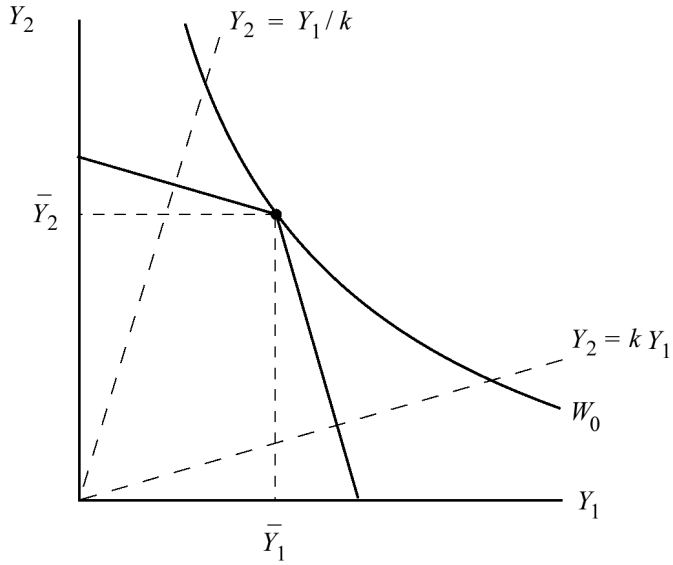
$$\overline{Y}_1 + \overline{Y_2} = \overline{Y}$$

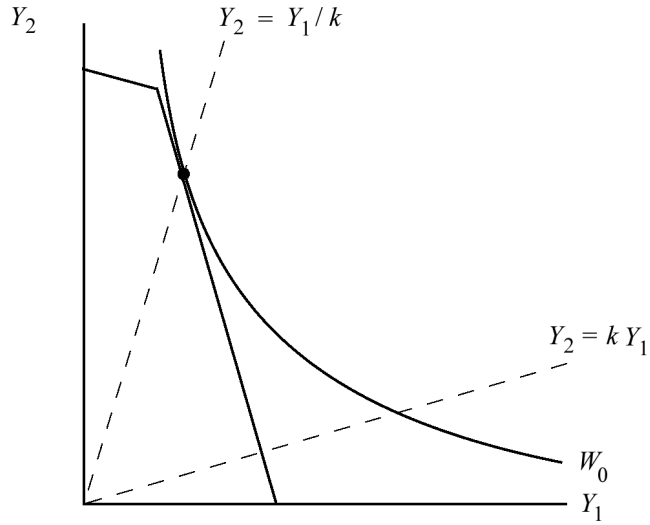
It follows that the tangency of the utility possibility frontier with a social indifference curve will occur at the same place for every such pair. That is, the optimal income distribution is the same for every pair $(\overline{Y}_1, \overline{Y}_2)$ satisfying this condition.

c) The utility possibility frontier has a kink at the initial income distribution. Taking a dollar from person 1 allows k dollars to be given to person 2, so the slope of the frontier

to the left of $(\overline{Y}_1, \overline{Y}_2)$ is $-k$. Taking a dollar from person 2 allows k dollars to be given to person 1, or equivalently, $1/k$ dollars must be taken from person 2 if person 1 is to get another dollar of income. Consequently, the frontier has slope $-1/k$ to the right of $(\overline{Y}_1, \overline{Y}_2)$.

The figure below shows a situation in which $k < \overline{Y}_2/\overline{Y}_1 < 1/k$. By (1), the slope of the social indifference curve passing through $(\overline{Y}_1, \overline{Y}_2)$ is $-(\overline{Y}_2/\overline{Y}_1)$ which, by assumption, is greater than $-1/k$ but less than $-k$. Since the indifference curve passing through $(\overline{Y}_1, \overline{Y}_2)$ is steeper than the flat part of the utility possibility frontier, welfare falls if income is transferred from person 1 to person 2. Since it is flatter than the steep part of the frontier, welfare falls if income is transferred from person 2 to person 1. Thus, the best policy is to do nothing. It can readily be shown that the best policy is to do nothing if $\overline{Y}_2/\overline{Y}_1$ is equal to k or $1/k$.



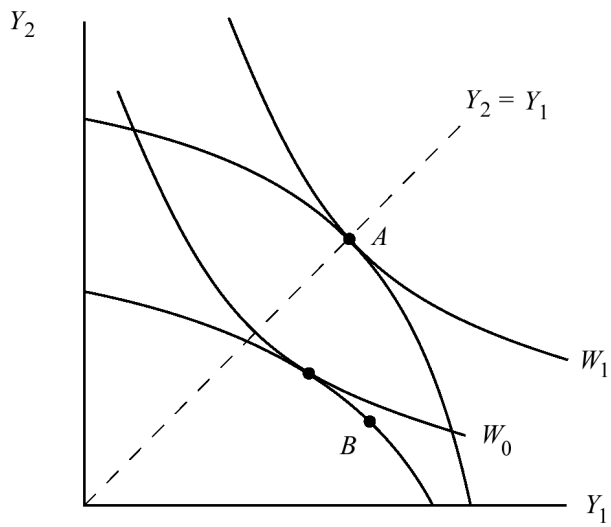


The second figure shows a case in which $\overline{Y}_2/\overline{Y}_1 > 1/k$. Now, the social indifference curve passing through $(\overline{Y}_1, \overline{Y}_2)$ is steeper than the steep section of of the utility possibility frontier, so a higher indifference curve can be reached by transferring income from person 2 to person 1. The optimal income distribution will be characterized by a tangency between an indifference curve and the frontier. Since the slope of this section of the frontier is $-1/k$, the slope of the indifference curve at the tangency must also be $-1/k$. By (1), the tangency occurs where Y_1 is equal to kY_2. A similar argument shows that if $\overline{Y}_2/\overline{Y}_1 < k$, the tangency occurs where Y_2 is equal to kY_1.

d) The utility possibility frontier in this case is bowed away from the origin. Two such

frontiers are shown in the last figure. In one case, $(\overline{Y}_1, \overline{Y}_2)$ occurs at A; and in the other, $(\overline{Y}_1, \overline{Y}_2)$ occurs at B. In both cases, the slope of the frontier is -1 at $(\overline{Y}_1, \overline{Y}_2)$ because no distribution is occurring. Taking successive dollars from one person yields every smaller gains for the other, so the frontier flattens to the left of $(\overline{Y}_1, \overline{Y}_2)$ and steepens to the right of it.

The incomes are equal at A. The social indifference curve at A therefore has a slope of -1, as does the frontier. Consequently, A is a point of tangency. If the initial income distribution is completely equal, the best policy is to stay there.



The initial income distribution at B is unequal, with Y_1 greater than Y_2. The slope of the indifference curve at B is therefore greater (*i.e.*, less negative) than -1. Since the slope of the frontier at that point is -1, the tangency must occur to the left. It cannot, however, occur so far to the left that the incomes are equalized. The slope of the frontier as it crosses the diagonal is greater than -1, while the slope of the indifference curve at that point is -1, so the tangency must occur to the right of this point. Thus, the optimal policy is to move to less unequal incomes, but not to complete equality.

Chapter 24: The Limits to Income Redistribution

Question 1

If the tax and the subsidy are too small to reduce the income gap to four dollars, every worker who has a choice of industries will work in the high-wage industry. Consequently, there will be three high-wage workers for every low-wage worker, and the government's tax revenue will equal its spending if:

$$3\tau = \sigma$$

Thus, (y_H, y_L) satisfy the conditions:

$$y_H = 10 - \tau$$

$$y_L = 4 + \sigma = 4 + 3\tau$$

A tax of 1/2 reduces the high-wage income to $9^1/_2$ and raises the low-wage income to $5^1/_2$. Since this tax reduces the income gap to four dollars, it is the highest tax at which three-quarters of the workers remain in the high-wage industry. Thus, the relationship between y_L and y_H in this case is:

$$y_L = 4 + 3\,(10 - y_H) = 34 - 3y_H \qquad 9^1/_2 \leq y_H \leq 10$$

When the income gap is exactly four dollars, the size of the tax and the subsidy depend upon the fraction of the workers who choose to remain in the high-wage industry. If this fraction is p, the government's tax revenue is equal to its spending when:

$$p\tau = (1 - p)\sigma$$

The possible pairs (y_H, y_L) now satisfy the conditions:

$$y_H = 10 - \tau$$

$$y_L = 4 + 3\left(\frac{p\tau}{1 - p}\right)$$

The requirement that the income gap be exactly four dollars implies:

$$y_H = y_L + 4$$

$$10 - \tau = 8 + 3\left(\frac{p\tau}{1 - p}\right)$$

$$\tau = 2(1 - p)$$

The last equation gives the tax that can be levied under each p. Substituting back into the income equations gives:

$$y_H = 8 + 2p$$

$$y_L = 4 + 2p$$

where $0 \leq p \leq 3/4$. In this case,

$$y_L = y_H - 4 \qquad 8 \leq y_H \leq 9^1/_2$$

An attempt to reduce the income gap below 4 cannot succeed, because every worker would leave the high-wage industry. There would be no-one left to pay the taxes.

Question 2 The incomes in the two industries are:

$$y_H = 4 + \frac{3}{p} - \tau \tag{1}$$

$$y_L = 4 + \sigma \tag{2}$$

The subsidy and the tax are connected by the requirement that the government spend exactly as much as it receives in taxes:

$$p\tau = (1-p)\sigma \tag{3}$$

The income gap will always be four dollars:

$$y_H = y_L + 4 \tag{4}$$

If the gap is initially less, workers will leave the high-wage industry, causing the wages there to be driven upwards until the four dollar gap is established. Conversely, if the gap is initially greater, workers will move into the high-wage industry, driving down the wages there until the income gap falls to four dollars. Substituting (3) into (2) to eliminate σ, and then substituting (1) and (2) into (4) yields, after some simplification,

$$\tau = (1-p)\left(\frac{3}{p} - 4\right) \qquad 0 \leq p \leq \frac{3}{4}$$

Consequently, the incomes associated with any given p are:

$$y_H = 11 - 4p$$

$$y_L = 7 - 4p$$

The best outcome for everyone is have as many people as possible in the high-wage industry. Not only do those people earn higher incomes, but the number of taxpayers is maximized and the number of subsidy recipients is minimized.

Chapter 25: Redistributing Income Through Tagging and Targeting

Question 1

The genuinely needy constitute half of the low income group, and four out of five of them have the required characteristics. It follows that 40% of the low income group is genuinely needy with the required characteristics and 10% is genuinely needy without the required characteristics. Since half of the low income group has the characteristics, 10% of the low income group must consist of people who are not genuinely needy but have the required characteristics. That leaves 40% of the population in the remaining group (low income without the required characteristics).

Every member of the low income population received a transfer under the old program. It follows that the fraction of the money accruing to members of each group was equal to the fraction of the low-income population constituted by that group.

Under the new program, the same amount of money is allocated to half as many people. Anyone who is still receiving a transfer receives twice as much. Thus, genuinely needy people with the required characteristics receive 80% of the funds (up from 40%); and people who are not genuinely needy but nonetheless have the required characteristics receive 20% of the funds (up from 10%).

People who are genuinely needy but do not have the required characteristics originally had 10% of the funding but now have none; and the people who are not genuinely needy and do not have the required characteristics originally had 40% of the funding but now have none.

It total, the fraction of the funding that goes to genuinely needy people has risen from 50% to 80%—but 20% of the genuinely needy have lost their funding entirely.

Question 2

a) Every person's budget constraint is:

$$c = 2h$$

Applying the usual procedure for finding the optimal commodity bundle gives:

$$h^* = 2\theta$$

$$c^* = 4\theta$$

The utility of a person of type θ is then 2θ.

Each person chooses his hours of work by equating the marginal disutility of an hour of work (h/θ) with the marginal utility of the consumption obtained by working another hour (w). Since neither of these factors is changed by a lump-sum tax or subsidy, a tax or subsidy would leave each person's hours of work unchanged. Consumption would change by the amount of the tax or subsidy, and utility would change by an equal amount.

b) For i), recall that if θ is uniformly distributed on the unit interval, the fraction of the population who have a θ smaller than $\overline{\theta}$ is $\overline{\theta}$, and the fraction of the population who have a θ larger than $\overline{\theta}$ is $1 - \overline{\theta}$. Consequently, the cost of the subsidy will be equal to the revenue from the tax if:

$$\overline{\theta}\sigma = \left(1 - \overline{\theta}\right)\tau \tag{1}$$

For ii), consider the utility of a person whose θ is greater than $1/4$. If he elects to pay the tax, his utility is:

$$u = 2\theta - \tau$$

because the tax has no effect on his hours of work. If he elects to receive the subsidy, he must reduce his earned income from 4θ to 1. He must therefore reduce his hours of work from 2θ to 1/2. His utility will therefore be:

$$\widehat{u} = 1 + \sigma - \frac{1}{8\theta}$$

The characteristic of $\overline{\theta}$ is that, at this value of θ, the utility of a person who pays the tax is the same as the utility of a person who receives the subsidy:

$$2\overline{\theta} - \tau = 1 + \sigma - \frac{1}{8\overline{\theta}} \tag{2}$$

Now, for iii), set $\overline{\theta}$ equal to 3/8 in (1) and (2), and then solve these two equations to find σ and τ. The solution is:

$$\sigma = 1/32$$

$$\tau = 5/96$$

c) The values of σ and τ will be found by solving a two equation system. One of the equations is (1), and the other will be found by equating u with a revised expression for $\widehat{u}$. An imposter must reduce his earned income from 4θ to 3/4. He must therefore reduce his hours of work from 2θ to 3/8. His utility will therefore be:

$$\widehat{u} = \frac{3}{4} + \sigma - \frac{1}{2\theta}\left(\frac{3}{8}\right)^2$$

A person of type $\overline{\theta}$ is indifferent between u and $\widehat{u}$:

$$2\overline{\theta} - \tau = \frac{3}{4} + \sigma - \frac{9}{128\overline{\theta}} \tag{3}$$

Setting $\overline{\theta}$ equal to 3/8 in (1) and (3), and then solving this system yields:

$$\sigma = 9/128$$

$$\tau = 15/128$$

The objective of this policy was to improve the welfare of those people whose θ is smaller than $1/4$. Not all of these people are affected by the policy in the same way. Those for whom θ is smaller than $3/16$ do not have to alter their hours of work to qualify for the subsidy, so their utility under the alternative program is:

$$u_1 = 2\theta + \frac{9}{128} \qquad \theta \leq \frac{3}{16}$$

those people whose θ lies between 3/16 and 1/4 must reduce their hours of work to qualify for the program, so:

$$u_1 = \frac{3}{4} + \sigma - \frac{1}{2\theta}\left(\frac{3}{8}\right)^2 \qquad \frac{3}{16} < \theta \leq \frac{1}{4}$$

Under the original program, the utility of any of these people would be:

$$u_0 = 2\theta + \frac{1}{32} \qquad \theta \leq \frac{1}{4}$$

Clearly, every person whose θ is smaller than $3/16$ prefers the alternative program to the new program. As for the remaining individuals, note that u_1 decreases as θ rises while u_0 rises as θ rises. It follows that the person who is least likely to prefer the alternative program to the new program is the one with the largest θ. The largest θ among the people in this group is 1/4, and evaluating the two utilities at this value of θ gives:

$$u_0|_{\theta=1/4} = 68/128$$

$$u_1|_{\theta=1/4} = 69/128$$

Thus, every person in the target group prefers the program under which earned income is constrained.