EC 502 Lecture 16: More on Real Business Cycles

Stephen J. Terry*

1 Adding Fiscal Policy via Government Spending

Let's start with the two-period RBC model with fixed labor supply from last time. We can summarize the equilibrium with the same equations as before, i.e.

$$U'(C_1) = \beta(1+r)U'(C_2) \quad \text{(HH Euler equation)}$$
$$r + \delta = \Pi_K(K_2; A_2, W_2) \quad \text{(Firm Euler equation)}$$
$$K_2 = (1-\delta)K_1 + I_1 \quad \text{(Capital accumulation equation)}$$
$$(1-\alpha)A_1 K_1^{\alpha} N^{-\alpha} = W_1 \quad \text{(Period 1 labor demand condition)}$$
$$(1-\alpha)A_2 K_2^{\alpha} N^{-\alpha} = W_2 \quad \text{(Period 2 labor demand condition)}$$
$$Y_1 = A_1 K_1^{\alpha} N^{1-\alpha} \quad \text{(Period 1 production function)}$$
$$Y_2 = A_2 K_2^{\alpha} N^{1-\alpha} \quad \text{(Period 2 production function)}$$
$$Y_1 = C_1 + I_1 \quad \text{(Period 1 resource constraint)}$$
$$Y_2 = C_2 - (1-\delta)K_2 \quad \text{(Period 2 resource constraint)}$$

Now let's add a government which chooses exogenously to spend in some amount G in period 1. The government funds its spending – which is assumed to be "useless" in the sense that it does not affect household utility in any way – by imposing lump-sum taxes $T = G$ on households in period 1. We call these taxes lump sum because they are taken as a fixed amount from earnings and do not distort or otherwise affect a household's incentives to save. This addition modifies the HH budget constraint to read

$$C_1 + S_1 = Y_0 + W_1 N - T.$$

You can show by taking the FOC of the HH utility maximization problem that the addition of government spending does not change the form of the household Euler equation, which reads

$$U'(C_1) = \beta(1+r)U'(C_2).$$

However, since government spending G is taken from output in period 1, the resource constraint must be modified to read

$$Y_1 = G + C_1 + I_1 \rightarrow C_1 = Y_1 - I_1 - G$$

The exact same arguments used to construct the unified Euler equation (2) from the last lecture can be repeated here to yield

$$U'(A_1 K_1^{\alpha} N^{1-\alpha} - K_2 + (1-\delta)K_1 - G) = \beta\left[\alpha A_2 K_2^{\alpha-1} N^{1-\alpha} + (1-\delta)\right] U'(A_2 K_2^{\alpha} N^{1-\alpha} + (1-\delta)K_2). \tag{1}$$

The only difference in this Euler equation is given by the presence of $-G$ on the LHS. Because

*These notes borrow heavily from notes by Adam Guren, Simon Gilchrist, and Francois Gourio.

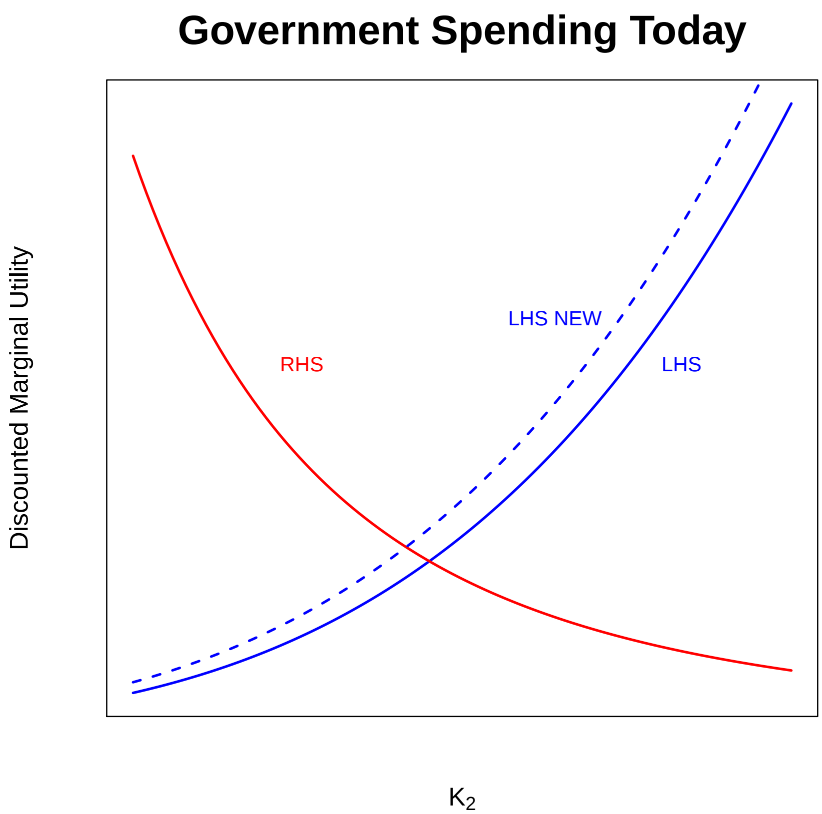

Figure 1: Increased Government Spending

$U'' < 0$, increased government spending shifts the LHS of the unified Euler equation (1) up as shown in Figure 1. Therefore, the equilibrium value of capital tomorrow K_2 declines. We conclude

- $I_1 = K_2 - (1-\delta)K_1$ declines because K_2 declines
- $Y_1 = A_1 K_1^\alpha N^{1-\alpha}$ is unchanged because A_1, K_1, and N are unchanged
- $Y_2 = A_2 K_2^\alpha N^{1-\alpha}$ declines because K_2 declines
- $W_1 = (1-\alpha)A_1 K_1^\alpha N^{-\alpha}$ is unchanged because A_1, K_1, and N are unchanged
- $W_2 = (1-\alpha)A_2 K_2^\alpha N^{-\alpha}$ declines because K_2 declines
- C_1 declines. This is not obvious at first, because a decline in I_1 could in principle increase consumption today, but Figure 1 demonstrates that at the new equilibrium value of K_2, we must have an increase in marginal utility $U'(C_1)$. Since $U'' < 0$, this implies that C_1 declines.
- $C_2 = Y_2 + (1-\delta)K_2 = A_2 K_2^\alpha N^{1-\alpha} + (1-\delta)K_2$ declines because K_2 declines.
- r increases. This follows because the interest rate – as shown in the last lecture – can be written $r = \alpha A_2 K_2^{\alpha-1} N^{1-\alpha} - \delta$. This expression is decreasing in K_2, and K_2 declines.

Intuitively, in this simple RBC framework, an increase in government spending leads to reduced consumption and investment today because G "crowds out" these activities. The result is a decline in capital, output, wages, and consumption tomorrow due to reduced investment today. Even though output levels do not change immediately in response to the government spending change, it can create a contraction in activity in the future.

Our simple two-period RBC model obviously provides a very grim view of government policy. There are several reasons for this:

- There is no productive use for government spending G.
- There are no "wealth effects" inducing poorer households to work more and raise output, since labor supply is fixed.
- Prices are flexible, so wages immediately adjust to lower capital tomorrow.

Later on, when we consider a New Keynesian business cycle model with nominal rigidities (i.e. sticky prices and wages), we will have more to say about a potential role for government policy to stabilize the economy. However, we can begin to discuss labor supply and wealth effects in the context of RBC models.

2 2-Period RBC Model with Labor Supply & Fixed Capital

Now, let's consider an alternative version of the basic RBC model, maintaining the assumption of two periods $t = 1, 2$ and certainty. However, instead of allowing investment and capital to adjust in response to changes in productivity, we will fix capital and allow labor supply to adjust.

2.1 Consumers

Following our discussion of labor supply in Lecture 12, households have a utility function in each period given by $U(C_t, L_t)$, where C_t is consumption in period t and L_t is leisure in period t. In each period, there is a total time budget of T units, so the implied labor is given by $N_t = T - L_t$. Households may save the amount S_1 in period 1 with a given interest rate r, starting from an initial wealth level Y_0. Labor in period 1 earns wages at rate W_1 and labor in period 2 earns wages at rate W_2. The utility maximization problem is given by

$$\max_{S_1, L_1, L_2} U(C_1, L_1) + \beta U(C_2, L_2)$$

$$C_1 = Y_0 + W_1(T - L_1) - S_1$$

$$C_2 = (1 + r)S_1 + W_2(T - L_2)$$

If we substitute the budget constraints into the objective we can see that the problem can be written

$$\max_{S_1, L_1, L_2} U\left[Y_0 + W_1(T - L_1) - S_1, L_1\right] + \beta U\left[(1 + r)S_1 + W_2(T - L_2), L_2\right],$$

and this problem has three first-order conditions wrt S_1, L_1, and L_2:

- Intertemporal Euler equation for S_1

$$U_C(C_1, L_1) = \beta(1 + r)U_C(C_2, L_2)$$

- Intratemporal Euler equation for L_1

$$W_1 U_C(C_1, L_1) = U_L(C_1, L_1)$$

- Intratemporal Euler equation for L_2

$$W_2 U_C(C_2, L_2) = U_L(C_2, L_2)$$

2.2 Firms

On the firm side, there is a fixed capital stock given by $K_t = K$ for $t = 1, 2$. Investment is equal to 0 for all periods, and depreciation is also set equal to 0. The production function is given by

$$Y_1 = A_1 K^\alpha N_1^{1-\alpha}$$

$$Y_2 = A_2 K^\alpha N_2^{1-\alpha}$$

Since $I_1 = I_2 = 0$, we simply have that $D_t = Y_t - W_t N_t$. In other words, the only decision the firm makes is the choice of labor demand N_1 and N_2. But since the choice of N_t only involves maximizing $Y_t - W_t N_t$, this is a static profit maximization problem in each period. The firm profit maximization, or equivalently labor demand, decision in each period can be written

$$N_1 = \arg\max_N A_1 K^\alpha N^{1-\alpha} - W_1 N$$

$$N_2 = \arg\max_N A_2 K^\alpha N^{1-\alpha} - W_2 N$$

Taking the first-order conditions with respect to N_t yields the standard profit maximization conditions for labor demand. These set the wage W_t equal to the MPL in period t, i.e.

$$W_t = (1-\alpha) A_t K^\alpha N_t^{-\alpha}.$$

2.3 General Equilibrium

General equilibrium in this economy is a set of wages W_1, W_2, an interest rate r, consumption C_1, C_2, labor demand N_1, N_2, and leisure choices L_1, L_2 such that

- **Households Optimize**: Given W_1, W_2, and r, households optimally choose consumption and leisure.
- **Firms Optimize**: Given W_1 and W_2 firms optimally choose labor demand in each period.
- **Markets Clear in Each Period**:
  - *Labor Markets Clear*: $N_1 = T - L_1$, $N_2 = T - L_1$
  - *Savings Market Clears*: $S_1 = 0$
  - *Goods Markets Clear* or *Resource Constraints Hold*: $Y_t = C_t$, $t = 1, 2$.

Notice that this equilibrium looks a little different than the one studied in Lecture 15. Here, there is no investment. The savings market must clear with $S_1 = 0$, and investment is omitted from the resource constraints. Also, there is a non-trivial leisure choice on the part of households. Given our analysis above, we can summarize this equilibrium involving the 11 variables W_1, W_2, r, Y_1, Y_2, L_1, L_2, N_1, N_2, C_1, and C_2 in terms of the 11 equations

$$U_C(C_1, L_1) = \beta(1 + r)U_C(C_2, L_2) \quad \text{(HH intertemporal Euler equation)}$$

$$W_1 U_C(C_1, L_1) = U_L(C_1, L_1) \quad \text{(HH Euler equation, Period 1 labor supply)}$$

$$W_2 U_C(C_2, L_2) = U_L(C_2, L_2) \quad \text{(HH Euler equation, Period 2 labor supply)}$$

$$N_1 = T - L_1 \quad \text{(Period 1 time budget constraint)}$$

$$N_2 = T - L_2 \quad \text{(Period 2 time budget constraint)}$$

$$Y_1 = A_1 K^\alpha N_1^{1-\alpha} \quad \text{(Period 1 production function)}$$

$$Y_2 = A_2 K^\alpha N_2^{1-\alpha} \quad \text{(Period 2 production function)}$$

$$W_1 = (1 - \alpha)A_1 K^\alpha N_1^{-\alpha} \quad \text{(Period 1 labor demand)}$$

$$W_2 = (1 - \alpha)A_2 K^\alpha N_2^{-\alpha} \quad \text{(Period 2 labor demand)}$$

$$C_1 = Y_1 \quad \text{(Period 1 resource constraint)}$$

$$C_2 = Y_2 \quad \text{(Period 2 resource constraint)}$$

2.4 Solving the Model

This foreboding system of equations can be simplified substantially. First, note that since there is no investment, the intertemporal Euler equation plays little role in the analysis. In this case, the interest rate will always adjust to set optimal savings equal to 0, but consumption is determined by the resource constraint $Y_t = C_t$ in each period. Now, do the following

- Substitute $N_t = T - L_t$ and $C_t = Y_t$ into the labor supply conditions and solve for W_t to obtain

$$W_t = \frac{U_L(Y_t, T - N_t)}{U_C(Y_t, T - N_t)}, \quad t = 1, 2$$

$$\to W_t = \frac{U_L(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}{U_C(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}, \quad t = 1, 2$$

- Eliminate W_t by using the labor demand curves to obtain

$$(1 - \alpha)A_t K^\alpha N_t^{-\alpha} = W_t = \frac{U_L(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}{U_C(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}, \quad t = 1, 2$$

$$\to (1 - \alpha)A_t K^\alpha N_t^{-\alpha} = \frac{U_L(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}{U_C(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}, \quad t = 1, 2$$

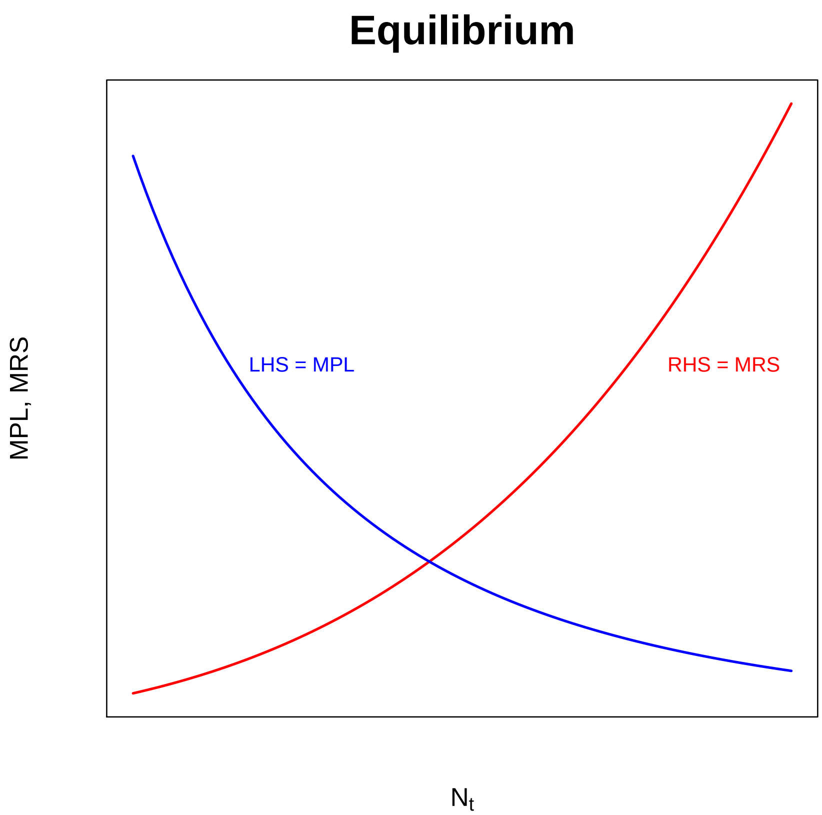

Figure 2: General Equilibrium Labor Supply

So in other words, the equilibrium of this economy boils down to equilibrium in the labor market in each period, determined by the two equations

$$(1-\alpha)A_1K^{\alpha}N_1^{-\alpha} = \frac{U_L(A_1K^{\alpha}N_1^{1-\alpha}, T-N_1)}{U_C(A_1K^{\alpha}N_1^{1-\alpha}, T-N_1)}$$

$$(1-\alpha)A_2K^{\alpha}N_2^{-\alpha} = \frac{U_L(A_2K^{\alpha}N_2^{1-\alpha}, T-N_2)}{U_C(A_2K^{\alpha}N_2^{1-\alpha}, T-N_2)}$$

In general, this is where the analysis must end. We can say definitively that the LHS of these equations – the MPL in period t – is decreasing in N_t. However, the RHS, the HH marginal rate of substitution between consumption and leisure, can be increasing or decreasing in N_t depending upon whether income or substitution effects of wage changes dominate. However, if we are willing to make some assumptions about the form of the utility function $U(C, L)$, we can go further. We analyze two special cases in the next subsections.

2.5 No Wealth Effects

Let's first consider an example assuming no wealth effects, which can be guaranteed with the utility function

$$U(C, L) = \log(C - v(T-L))$$

- The function $v(N) = v(T - L)$ governs the disutility of labor.
- We assume $v'(N) > 0$, $v''(N) > 0$, so households dislike labor at an increasing rate.
- We have that
$$U_C(C,L) = \frac{1}{C - v(N)}, \quad U_L(C,L) = \frac{v'(N)}{C - v(N)}$$

So given this functional form for utility we can write the household marginal rate of substitution between consumption and leisure as

$$\frac{U_L(C,L)}{U_C(C,L)} = v'(N).$$

Therefore, the two labor market equilibrium conditions can be written

$$(1-\alpha)A_1 K^\alpha N_1^{-\alpha} = v'(N_1)$$

$$(1-\alpha)A_2 K^\alpha N_2^{-\alpha} = v'(N_2)$$

As we noted earlier, the LHS is simply the MPL in each period, which is declining in N_t. However, in this case without wealth effects the RHS is guaranteed to be increasing in N_t since $v''(N_t) > 0$. Intuitively, households always increase their labor supply in response to higher wages if only substitution effects are present, so the wage – and hence the RHS – is equal to an increasing function of labor. In this case, the LHS and RHS of the equilibrium conditions pinning down labor supply – or hours worked – will look something like Figure 2.

A Specific Functional Form

To go further, let's pick a specific function form $v(N) = \frac{N^{1+\phi}}{1+\phi}$, with $\phi > 0$, for the function $v(N)$. This satisfies $v'(N) = N^\phi > 0$ and $v''(N) = \phi N^{\phi-1} > 0$, as desired. With this functional form, the value $\frac{1}{\phi}$ is known as the **labor supply elasticity**, since the HH labor supply condition can be written $N_t^\phi = W_t \rightarrow N_t = W_t^{\frac{1}{\phi}}$. It turns out that we can now solve for the values of all of the endogenous variables in this model:

- **Labor** N_t
We have that the unified labor market equilibrium conditions can be written
$$(1-\alpha)A_t K^\alpha N_t^{-\alpha} = N_t^\phi$$
$$N_t = (1-\alpha)^{\frac{1}{\alpha+\phi}} A_t^{\frac{1}{\alpha+\phi}} K^{\frac{\alpha}{\alpha+\phi}}$$
- **Leisure** L_t
Trivially, we have that leisure in each period is given by $L_t = 1 - N_t$, where N_t is pinned down above.
- **Output** Y_t and **Consumption** C_t
Output in each period and consumption are given by the production function and resource constraint.
$$Y_t = C_t = A_t K^\alpha N_t^{1-\alpha} = (1-\alpha)^{\frac{1-\alpha}{\alpha+\phi}} A_t^{\frac{1+\phi}{\alpha+\phi}} K_t^{\frac{\alpha(1+\phi)}{\alpha+\phi}}$$

- **Wages** W_t
  Wages in each period are given by the HH labor supply condition

$$W_t = N_t^{\phi} = (1-\alpha)^{\frac{\phi}{\alpha+\phi}} A_t^{\frac{\phi}{\alpha+\phi}} K^{\frac{\alpha\phi}{\alpha+\phi}}$$

- **Interest Rate** r
  Given the values of C_1, C_2, L_1, and L_2 determined above, we have from the HH intertemporal Euler equation that the interest rate is determined by

$$r = \frac{1}{\beta}\frac{U_C(C_1, L_1)}{U_C(C_2, L_2)} - 1$$

Without wealth effects, these equations imply that the economy responds with a boom after an increase in productivity A_t. In particular, by taking logs for the formulas for output Y_t above, we can see that

$$\frac{\partial \log Y_t}{\partial \log A_t} = \frac{1+\phi}{\alpha+\phi} > 0$$
$$\frac{\partial \log N_t}{\partial \log A_t} = \frac{1}{\alpha+\phi} > 0$$

In other words, in general equilibrium the elasticities of output and labor with respect to productivity are positive. After a positive TFP shock, the marginal product of labor increases. This causes firms to demand more labor, leading to increased wages and higher labor supply on the part of households. Because Y_t, C_t, and N_t move together, we see that a TFP shock in this case again leads to business cycle co-movement.

2.6 Wealth Effects

Now, let's consider a second example with wealth effects on labor supply present. In particular, let's use the period utility function

$$U(C, L) = \log(C) - v(T - L),$$

where $v'(N) > 0$ and $v''(N) > 0$ as before. Let's also go ahead and use the same form for $v(N)$ as before, with $v(N) = \frac{N^{1+\phi}}{1+\phi}$ where $\phi > 0$. We have that $U_C(C, L) = \frac{1}{C}, \quad U_L(C, L) = v'(T - L) = v'(N) = N^{\phi}$ so that the marginal rate of substitution is given by

$$\frac{U_L(C, L)}{U_C(C, L)} = v'(N)C = N^{\phi}C.$$

The reason we say that this utility function exhibits wealth effects is that the HH labor supply condition can be written

$$N_t^{\phi} C_t = W_t \rightarrow N_t = \left(\frac{W_t}{C_t}\right)^{\frac{1}{\phi}}.$$

In response to an increase in the wage W_t, HH's would like to work more, represented by what we call the substitution effect in the numerator. However, since the wage also increases the consumption level C_t today, the marginal utility of consumption declines and households would like to work less. This is the income or wealth effect represented by the C_t in the denominator. With this function form, the unified labor market equilibrium conditions can be written

$$(1-\alpha)A_t K^\alpha N_t^{-\alpha} = \frac{U_L(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}{U_C(A_t K^\alpha N_t^{1-\alpha}, T - N_t)}.$$

But noting that $C_t = Y_t$ and given the formula for the marginal rate of substitution above, we have

$$(1-\alpha)A_t K^\alpha N_t^{-\alpha} = N_t^\phi A_t K^\alpha N_t^{1-\alpha}$$

$$N_t = (1-\alpha)^{\frac{1}{1+\phi}}.$$

In this case, labor is always fixed! As noted above, an increase in productivity A_t leads to two offsetting effects:

- **Substitution Effect**: If A_t increases, W_t increases and HH's want to work more.
- **Income Effect**: If A_t increases, C_t also increases, so HH's want to work less.

With log utility, these two forces exactly offset and labor does not respond to productivity shocks. However, we still have that

$$C_t = Y_t = A_t K^\alpha N_t^{1-\alpha} = A_t K^\alpha (1-\alpha)^{\frac{1-\alpha}{1+\phi}},$$

so in response to a productivity shock in this environment both Y_t and C_t comove.