1. Assume that the underlying asset price *S* follows geometric Brownian motion:

,

where *r* and are constants. Find the explicit solution for the value of a European option with payoff



and expiry at time *T*.

1. In the continuous dividend-paying model, the underlying asset price process is given by

dSt/St = dt + dWt ,

and at the dividend structure is given by

dD(t) = Sdt.

The pricing function *F*(t, s) of the claim  solves the boundary value problem:

.

Find the arbitrage free price, *F*(t, s), for the claim X = ln(ST)2 .

1. Let X(t) be the spot exchange rate between USD and Euro, namely, $1 is worth X(t) Euros. We assume the following dynamics:

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the price process of the risk-free domestics money account

,

and the price process of the risk-free foreign money account

.

The pricing function *F*(t, *x*) of the claim  solves the boundary value problem:

.

Compute the price of the European put with exercise exchange rate K = .70 Euros/USD and the time to expiry is 6 months. (Assume *rd*= 5%, and *rf*= 2%. Today’s exchange rate $1 = .78 Euros.)

1. The price of a European call is given by



where  and .

a. Find the price of the option if S =$10, t = 0, T = 3 months, K = $12, r = 10%, 

b. Verify that  satisfies the Black-Sholes differential equation:



and the price at the maturity is

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1. Exercise 9.10 on page 136.
2. For the Hull-White model

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find the SDE that the instantaneous forward rate *f(t,T)* satisfies.

1. Find the implied volatility of the following European call. The call has four months until expiry and an exercise price of $100. The call is worth $6.51 and the underlying trades at $101.50. Assume the risk free interest rate is 8% per annum.

[Hint: try different values of You may use the bisection method. 