Convergence Analyses of Timing and Carrier Recovery Loops for a Digital Communication Channel with an Adaptive Equalizer

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Abstract — In this paper, convergence behaviors of timing and carrier recovery loops, when an adaptive equalizer is used together with decision-directed types of timing and carrier recovery loops in tracking mode for a digital communication system, is discussed. The phenomenon of getting biased in timing or carrier phase when convergence parameters of an adaptive equalizer, a timing recovery loop, and a carrier recovery loop are not selected properly is investigated for a digital communication channel.

I. INTRODUCTION

In recent years, adaptive equalization, timing recovery, and carrier recovery have been intensively investigated to improve the performance of digital communication systems [1], [2]. However, most of studies for adaptive equalization were performed by assuming that both timing and carrier recovery loops were working perfectly without any timing jitter or phase jitter [2], [3]. Also, decision-directed types of timing recovery loops or carrier recovery loops were investigated by assuming that the other parts in the receiver were working correctly [1], [4]. However, in a practical situation, coefficients of an adaptive equalizer must be updated even with the data poorly sampled in timing phase or incorrectly demodulated in carrier phase [5], [6]. Then, is it possible to make the coefficients of the adaptive equalizer, timing phase, and carrier phase converge correctly even with the error signal caused by either incorrect phase or channel variation?

In this paper, convergence behaviors of timing and carrier recovery loops when decision-directed types of timing and carrier recovery loops are working together with an adaptive equalizer in a digital communication system are discussed. Since adaptive algorithms for equalizers, decision-directed timing recovery loops, and decision-directed carrier recovery loops are all derived under the same criterion (MSE), the error caused by any source such as channel variation, timing jitter, and frequency offset can affect the performances of all three parts. In order to investigate the relationship among timing recovery, carrier recovery, and adaptive equalization, the basic equation for each part is first discussed by taking into account these error sources in Section II. Then, convergence behavior of timing (carrier) phase when a decision-directed timing (carrier) recovery loop is used together with an adaptive equalizer is discussed. Discussion on the relationship between the timing recovery loop and the carrier recovery loop is followed. Conclusion is made in Section III.

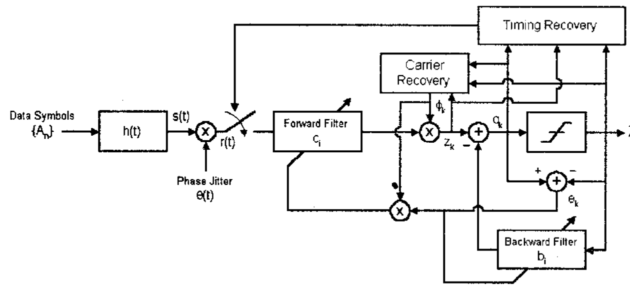

Fig. 1. A block diagram of a typical digital communication.

II. RELATIONSHIP AMONG TIMING RECOVERY, CARRIER RECOVERY, AND ADAPTIVE EQUALIZATION

Most of the receivers currently used for digital communication systems consist of a front-end filter, demodulator, timing recovery loop, carrier recovery loop, adaptive equalizer, and symbol-decision device. A typical receiver for a digital communication system with a decision-feedback equalizer (DFE), a decision-directed timing recovery loop, and a decision-directed carrier recovery loop are shown in Fig. 1. For the sake of simplicity, it is assumed that the front-end filtering was performed beforehand, and that preliminary demodulation by a local oscillator with a frequency f_c was carried out before equalization [7].

The received signal, $x(t)$, for a bandpass channel with frequency offset or phase jitter is expressed as

$$x(t) = \exp(j(2\pi f_c t + \theta(t)))s(t) = \exp(j(2\pi f_c t + \theta(t)))\sum_n A_n h(t - nT), \tag{1}$$

where T denotes the symbol interval. A_n represents the transmitted symbol, and $h(t)$ represents the impulse response of the system consisting of a transmitter filter, a channel, and a receiver filter. $\theta(t)$ is defined by

$$\theta(t) = \theta + 2\pi\Delta t + \varphi(t), \tag{2}$$

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where θ, Δ, $\varphi(t)$ represent a fixed phase shift, a fixed frequency offset, and a random waveform that models the phase jitter, respectively. The peak magnitude of the waveform, $\varphi(t)$, is known to be less than 10° for voiceband telephone channels [5]. After preliminary demodulation with a frequency f_c, the sampled version of the received signal at times $kT+\tau_k$ is given by

$$r_k(\tau_k) = r(kT+\tau_k) = \exp(j\theta_k)\sum_n A_n h(kT+\tau_k - nT). \tag{3}$$

After passing through the forward filter of a DFE, the signal compensated by a carrier recovery loop with ϕ_k is expressed as

$$z_k(\tau_k,\phi_k) = \exp(j(\theta_k - \phi_k))\sum_{i=0}^{N-1} c_i(k)s(kT+\tau_k - iT), \tag{4}$$

where $c_i(k)$ denotes the i-th tap of the forward filter at time k. As shown in Fig.1, the input signal of a decision device is given by

$$q_k(\tau_k,\phi_k) = z_k(\tau_k,\phi_k) - \sum_{i=1}^{M} b_i(k)\hat{A}_{k-i}, \tag{5}$$

where $b_i(k)$ denotes the i-th tap of the backward filter at time k. $\hat{A}_{k-i}$ represents the input symbol estimated by the decision device. The MSE criterion widely used for adaptive equalizer and decision-directed types of timing and carrier recovery loops is defined by

$$E[|e_k(\tau_k,\phi_k)|^2] = E[|q_k(\tau_k,\phi_k) - \hat{A}_k|^2]. \tag{6}$$

When τ_k and $\theta_k - \phi_k$ are given by constants, τ and $\theta-\phi$, respectively, the MSE criterion in (6) can be expressed in the frequency domain as follows:

$$E[|e(\tau,\phi)|^2] = T\int_{-1/2T}^{1/2T}|P(f)-1|^2 df, \tag{7}$$

where $P(f)$ represents the frequency response of the equivalent discrete-time system consisting of the channel $h(t)$, forward and backward filter of DFE, carrier recovery loop, and timing recovery loop, i.e.,

$$\begin{aligned} P(f) &= C(f)(1/T)\sum_{m=-\infty}^{\infty} H(f-m/T) \\ &\cdot \exp(j[2\pi(f-m/T)\tau + (\theta-\phi)]) - B(f), \end{aligned} \tag{8}$$

where $H(f)$ represents the Fourier transform of $h(t)$. Also, $C(f)$ and $B(f)$ represent the Fourier transforms of forward filter, c_i and backward filter, b_i, of the DFE, respectively.

The MSE criterion has been widely used for deriving adaptive algorithms for equalizers. The same criterion has also been used for tracking sampling and carrier phases in the decision-directed types of timing and carrier recovery loops. By taking into account the timing and carrier phase errors, the LMS algorithm for the DFE can be written as

$$c_i(k+1) = c_i(k) - (\mu/2)\hat{\nabla}_{c_i(k)}, \quad i=0,1,\cdots,N-1, \tag{9}$$

$$b_i(k+1) = b_i(k) - (\mu/2)\hat{\nabla}_{b_i(k)}, \quad i=1,2,\cdots,M, \tag{10}$$

where

$$\begin{aligned} \hat{\nabla}_{c_i(k)} &= 2e_k^*(\tau_k,\phi_k)s(kT+\tau_k - iT) \\ &\cdot \exp(j(\theta_k - \phi_k)), \quad i=0,1,\cdots,N-1, \end{aligned} \tag{11}$$

$$\hat{\nabla}_{b_i(k)} = -2e_k^*(\tau_k,\phi_k)\hat{A}_{k-i}, \quad i=1,2,\cdots,M. \tag{12}$$

Here, the step-size parameter, μ, determines a tradeoff between the convergence speed and the error variance of the equalizer. Under the assumption of Nyquist-rate sampling, the approximate MMSE technique for tracking sampling phase in a decision-directed timing recovery loop can be written by

$$\tau_{k+1} = \tau_k - (\alpha/2)\hat{\nabla}_{\tau_k}, \tag{13}$$

where

$$\hat{\nabla}_{\tau_k} \approx 2\,\mathrm{Re}\{e_k^*(\tau_k,\phi_k)[z_k(\tau_k,\phi_k) - z_{k-2}(\tau_k,\phi_k)]\}. \tag{14}$$

Here, α denotes a timing convergence constant. Also, the decision-directed carrier recovery technique used in conjunction with an adaptive equalizer and a timing recovery loop can be written by

$$\phi_{k+1} = \phi_k - (\alpha/2)\hat{\nabla}_{\phi_k}, \tag{15}$$

where

$$\begin{aligned} \hat{\nabla}_{\phi_k} &= 2\,\mathrm{Re}\{e_k^*(\tau_k,\phi_k)\cdot(-j)z_k(\tau_k,\phi_k)\} \\ &= 2\,\mathrm{Im}\{e_k^*(\tau_k,\phi_k)z_k(\tau_k,\phi_k)\}. \end{aligned} \tag{16}$$

Here, K_L denotes a loop filter gain of the 1^{st}-order PLL.

As indicated by (11), (12), (14), (16), the error signal is caused not only by the equalizer whose coefficients are not set to optimal values, but also the imperfect timing and carrier phases. That is, the error signal created by any causes will force the coefficients of an adaptive equalizer, timing and carrier phases to move from current values to new ones

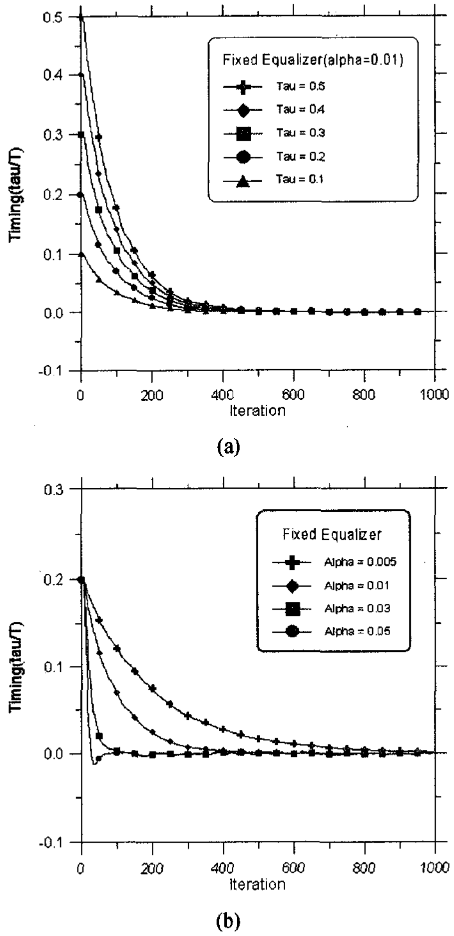


Fig. 2. Timing recovery behavior when an optimal fixed equalizer and a timing recovery loop are used together.

(a) When timing disturbance (τ /T) is present.
(b) When timing convergence constant (α) is varied.

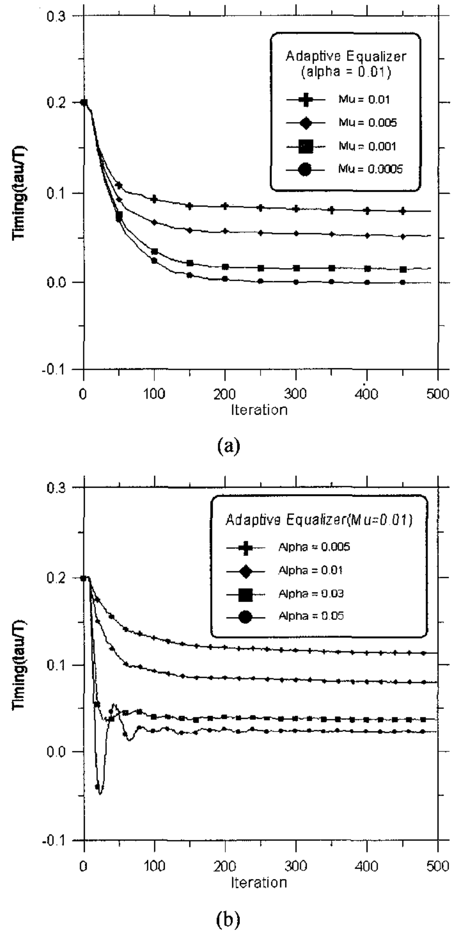


Fig. 3. Timing recovery behavior when an adaptive equalizer and a timing recovery device are used together.

(a) When the parameter (μ) of adaptive equalizer is changed (α =0.01).
(b) When the timing convergence constant (α) is varied (μ =0.01).

in the direction of minimizing the MSE, which may lead to biased ones. In order to illustrate the phenomenon occurring in this situation, a typical telephone channel with a QPSK input is applied to the digital communication system, shown in Fig. 1. Fig. 2 shows timing recovery behavior when tap coefficients of the equalizer are set to optimal values and perfect carrier recovery is assumed ($\theta(t) = 0^\circ$). Note that the convergence characteristics with $\alpha = 0.01$ are similar for all different scales of perturbations, $\tau / T = 0.1 \sim 0.5$. From Fig. 2(b), one can see that, as α increases, the convergence time decreases at the expense of increased variance in timing phase, as expected. Next, let's consider the case where timing recovery scheme is used with an adaptive DFE. From Fig. 3, one can see that timing phases converge to correct values at the expense of larger variance of timing phase when the timing recovery constant, α , is relatively greater than the step size, μ . The amount of bias in timing phase varies depending on the magnitude of perturbation, α , and μ , when timing recovery loop is used together with an adaptive DFE.

The phenomenon of getting bias in timing phase depending on the values α and μ can be investigated by utilizing the MSE equations in (6) or (7). Since there exist filter coefficients which minimize the MSE for any given timing phase, τ , the optimal coefficients of the adaptive DFE and the corresponding MSE will be different for a given timing phase. Fig. 4 shows a plot of MSE when an adaptive DFE and a timing recovery loop are used together. The value at the point (τ_1, τ_2) represents the MSE's when the timing phase is given by τ_2 and the filter coefficients are set to the values minimizing MSE for τ_1 . Let's take a look at the case where the perturbation in timing phase, τ =0.2, occurs for this channel that has been working in ideal condition. At this instant, the value of MSE will be abruptly increased from the one at (0, 0) to the one at (0.2, 0) since the filter coefficients

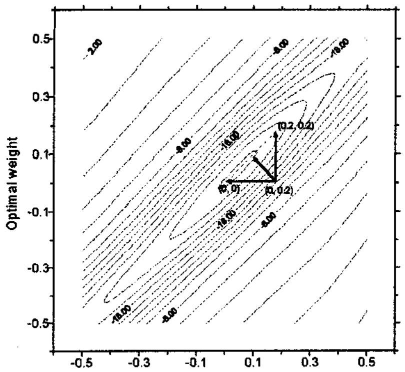
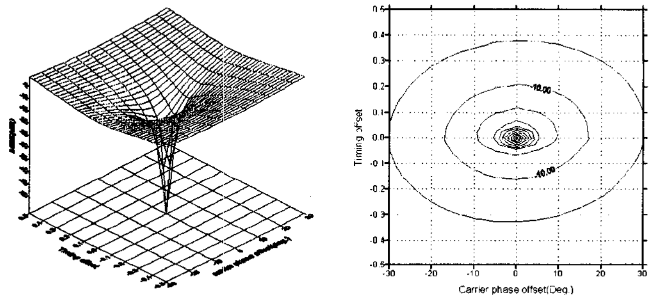


Fig. 4. Contour plot of MSE when adaptive DFE and timing recovery device are used together.



(a) (b)

Fig. 5. MSE when a timing recovery loop and a carrier recovery loop are used together with a fixed equalizer.
(a) 3-dimensional plot of MSE. (b) Contour plot of MSE.

of the adaptive filter cannot be changed instantaneously. When the MSE is increased by the perturbation in timing phase, the updating algorithms for adaptive equalizer and timing recovery loop will be processed in the direction which minimizes the MSE. However, the way in which the adaptive equalizer and the timing recovery loop converge to minimum MSE will be different for a given μ and α. Here, the step size, μ, determines the convergence speed of the equalizer to the point of minimum MSE for a given value of τ, and α controls the speed of timing recovery with the given coefficients of the equalizer. Thus, the actual direction becomes the sum of these two vectors. For the previous case where the starting point is set at (0.2, 0) due to the perturbation, it will converge to the point of (0, 0), resulting in an unbiased timing phase, if the value of α is relatively greater than that of μ. That is, the direction of the sum of two vectors in this case will be the neighborhood of the center in Fig. 4. On the other hand, if the value of μ is relatively greater than the one of α, it will converge to the point of (0.2, 0.2), resulting in a biased timing phase. In other words, if the value of μ is relatively greater than the one of α, the coefficients of the adaptive filter converge fast to the corresponding values of the point (0.2, 0.2), where the MSE is minimum for τ =0.2, before the timing phase is recovered. Then, the convergence rate of the equalizer and the timing recovery loop, pointing to the global minimum, (0, 0), will be extremely slow due to negligible difference between MSE's.

The relationship between the timing recovery loop and the carrier recovery loop when they are used together with a fixed equalizer can be shown by following the same approach. From Fig. 5, one can see that timing and carrier phases converge to correct values irrespective of given values of parameters, α and K_L. As opposed to the previous results, there exists only one global minimum point for MSE, implying that the carrier recovery loop and the timing recovery loop work independently.

III. Conclusion

In this paper, convergence behavior of timing and carrier recovery loops, when an adaptive equalizer is used together with decision-directed types of timing and carrier recovery loops in tracking mode for a digital communication channel, is discussed. The phenomenon of getting biased in timing phase or carrier phase when the convergence parameters of an adaptive equalizer, a timing recovery loop, and a carrier recovery loop are not selected properly is investigated for a typical digital communication channel. It is also shown that adaptive equalization and carrier recovery are closely related such that a small phase jitter can be recovered by using an adaptive equalizer, while the interaction between the carrier recovery loop and the timing recovery loop is orthogonal.

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