Paper Code: 715189
Algebra and Discrete Mathematics
Lecturer: Ji Ruan, Kate Lee
Assignment 2
Due 4:00pm, Monday 22 September 2014

Name .... **ID number**..........................

Question	Marks Possible	Marks Given
1	16	
2	20	
3	15	
4	24	
Total	75	

Instructions:

Please attach this sheet to the front of your assignment.

The assignment must be handed in before 4.00 p.m. on **Monday 22 September 2014**.

- For the students in City Campus, you need to put the assignment answers in the assignment box at the reception of level 1 WT building (2-14 Wakefield St).
- For the students in South Campus, you can also submit yours to the same assignment box (clearly stating that you are from South Campus) or arrange with your lecturer Kate Lee.

This is an individual assignment. The point of the assignment is for you to go through the process of discovery for yourself. Copying someone else's work will not achieve this. Plagiarism has occurred where a person effectively and without acknowledgement presents as their own work the work of others. That may include published material, such as books, newspapers, lecture notes or handouts, material from the www or other students' written work. That work also includes computer output.

The School of Computing and Mathematical Sciences regards any act of cheating including plagiarism, unauthorised collaboration and theft of another student's work most seriously. Any such act will result in a mark of zero being given for this part of the assessment and may lead to disciplinary action.

Please sign to signify that you understand what this means, and that the assignment is your own work.

Signature: ....

Question 1 (16 marks) *Answer the following questions and show your work:*

(a) *Let f be a function $f : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ such that $f(n) = (2n, n + 3)$. Verify whether this function is 1-1 and whether it is onto.*

(b) *Let f be a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $f(x, y, z) = xyz$. Verify whether this function is 1-1 and whether it is onto.*

(c) *Prove that the function $f : \mathbb{R} - \{2\} \to \mathbb{R} - \{5\}$ defined by $f(x) = \frac{5x+1}{x-2}$ is a bijection.*

(d) *Consider the functions $f : \mathbb{R} \to \mathbb{R}, g : \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined as $f(x) = \frac{1}{x^2+1}$ and $g(x) = (3x, x^2)$. Find the formulas for $f \circ f$ and $g \circ f$.*

Question 2 (20 marks) *Use* **truth tables** *to show the following:*

(a) *whether $\neg p \wedge \neg(p \to q)$ is a tautology, a contradiction or neither.*

(b) *whether $((p \to q) \wedge (p \to r)) \to (p \to (q \wedge r))$ is a tautology, a contradiction or neither.*

(c) *$\neg(\neg p \wedge q)$ and $q \to p$ are logically equivalent.*

(d) *$(p \to (q \to r))$ and $(q \to (p \to r))$ are logically equivalent.*

Question 3 (15 marks) *Use* **laws of logic (algebraic version)** *to show the following equivalences (clearly indicate which law you use in each step):*

(a) $\overline{(\overline{p} \wedge q)} \equiv (q \to p)$.

(b) $(p \to (q \to r)) \equiv (q \to (p \to r))$

(c) $((p \to r) \vee (q \to r)) \equiv ((p \wedge q) \to r)$

Question 4 (24 marks) *Prove the following using the method suggested:*

(a) *Prove the following either by direct proof or by contraposition:*
Let $a \in \mathbb{Z}$, if $a \equiv 1 \pmod 5$, then $a^2 \equiv 1 \pmod 5$.

(b) *Prove the following by contradiction:*
Suppose $a, b \in \mathbb{Z}$. If $4|(a^2 + b^2)$, then a and b are not both odd.

(c) *Disprove the following by counterexamples:*

- *For every natural number n, the integer $n^2 + 17n + 17$ is prime.*
- *Let A, B and C be sets. If $A \times C = B \times C$, then $A = B$.*

(d) *Prove the following by cases: For all $n \in \mathbb{Z}$, $n^2 + 3n + 4$ is even.*

(e) *Prove the following by induction:*

$$1^2 + 3^2 + 5^2 + \cdots + (2n - 1)^2 = \frac{n}{3}(2n - 1)(2n + 1)$$