Properties of Matrix Multiplication

Associative Property

1 Matrix multiplication is associative. That is, for any three matrices A, B, and C of appropriate size, the following matrix multiplication equality is true:

$$(AB)C = A(BC).$$

Demonstrate this property for the following three matrices:

$$A = \begin{pmatrix} 4 & 3 & 1 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & -5 \\ 3 & 1 \\ 5 & 0 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 6 & 7 \end{pmatrix}.$$

(i) $AB = \underline{\ ?\ }$ (ii) $(AB)C = \underline{\ ?\ }$

(iii) $BC = \underline{\ ?\ }$ (iv) $A(BC) = \underline{\ ?\ }$

(v) Your answers in (ii) and (iv) should agree.

Distributive Property

2 Matrix multiplication is distributive. That is, for any four matrices A, B, C and D of appropriate size, the following two matrix multiplication equalities are true:

$$A(B+C) = AB + AC \qquad \text{and} \qquad (B+C)D = BD + CD.$$

Demonstrate this property for the following four matrices:

$$A = \begin{pmatrix} -2 & 7 & 1 \\ 3 & -5 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix}, \quad \text{and} \quad D = \begin{pmatrix} 3 & -4 \\ 5 & 0 \\ -11 & 2 \end{pmatrix}$$

(o) $(B+C) = \underline{\ ?\ }$

(i) $A(B+C) = \underline{\ ?\ }$

(ii) $AB = \underline{\ ?\ }$ (iii) $AC = \underline{\ ?\ }$ (iv) $AB + AC = \underline{\ ?\ }$

(v) Your answers in (i) and (iv) should agree.

(vi) $(B+C)D = \underline{\ ?\ }$

(vii) $BD = \underline{\ ?\ }$ (viii) $CD = \underline{\ ?\ }$ (ix) $BD + CD = \underline{\ ?\ }$

(x) Your answers in (vi) and (ix) should agree.

Commutative Property

3 Matrix multiplication is not commutative, in general. That is, for any two matrices *A* and *B*, it does *not necessarily* follow that: $BA = AB$. Demonstrate this fact for the following matrices:

$$A = \begin{pmatrix} -2 & 7 & 1 \\ 3 & -5 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix}, \quad \text{and} \quad D = \begin{pmatrix} 3 & -4 \\ 5 & 0 \\ -11 & 2 \end{pmatrix}$$

(i) $AD = \underline{?}$ (ii) $DA = \underline{?}$ (iii) $AD \overset{?}{=} DA$

(iv) $BC = \underline{?}$ (v) $CB = \underline{?}$ (vi) $BC \overset{?}{=} CB$

(vii) $AB = \underline{?}$ (viii) $BA = \underline{?}$ (ix) $AB \overset{?}{=} BA$

4 Double-check all your computations in #3 using Maple. Zero points if the Maple check does not agree with the hand-calculation. So, if your Maple check does not agree with your hand-computation, then find your error and correct it. Provide a computer printout of both the commands and the output for each check.

Scalar Multiplication Property

5 Any scalar can always be factored out of matrix multiplication. In other words, for any two matrices *A* and *B* of appropriate size and for any scalar (number) c, the following is true:

$$c(AB) = (cA)B = A(cB).$$

Demonstrate this fact for the following matrices:

$$A = \begin{pmatrix} -2 & 7 & 1 \\ 3 & -5 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & -4 \\ 5 & 0 \\ -11 & 2 \end{pmatrix}$$

(i) $AB = \underline{?}$ (ii) $-3(AB) = \underline{?}$

(iii) $(-3A)B = \underline{?}$ (iv) $= A(-3B) = \underline{?}$

(v) Your answers in (ii), (iii) and (iv) should agree.