CSCI 2824 Discrete Structures
Instructor: Hoenigman
Assignment 1

Due Date: 09/05/2013 (Thursday, at the beginning of class).

Problem 1 (35 points)

For this problem, you are asked to write down a **recurrence relation** and the **closed form** for each of the sequences described below. In each case the indices n are natural numbers and thus $n \geq 0$.

1. $a_n = 1, 2, 4, 8, 16, \ldots$ (the sequence of all powers of 2).
2. $b_n = 1, 3, 2, 9, 4, 27, 8, 81, \ldots$ (altenating powers of 2 and 3).
3. $c_n = 0, 1, 3, 6, 10, 15, \ldots$ (Hint: look at the differences between successive elements. That should immediately suggest a recurrence.)
4. $d_n = 1, 0, 1, 0, 1, 0, 1, 0, \ldots$ (sequence of alternating 1s and 0s).
5. $e_n = 1, 1, 0, 0, 1, 1, 0, 0, \ldots$ (block of two ones, followed by a block of two zeros, followed by a block of two ones ...)

Problem 2 (35 points)

Write down recurrence equations for the sequences with the closed forms and summations given below. In each case assume $n \in \mathbb{N}$.

1. $p_n = 2^{n+2}$.
2. $q_n = n!$ (note that $0! = 1$, by definition)
3. $r_n = 2n^2 - 3n + 5$.
4. $s_n = \begin{cases} \frac{n}{2} & n \text{ is even} \\ \frac{n+1}{2} & n \text{ is odd} \end{cases}$
5. $t_n = \frac{1}{n+1}$.
6. $u_n = \sum_{j=1}^{n} (2j + 1)$
7. $v_n = \prod_{j=1}^{n} 2^n$

Problem 3 (30 points)

Let s_n be a sequence for $n \in \mathbb{N}$. Its *first difference sequence* d_n is defined by $d_n = s_{n+1} - s_n$. Answer the following questions about the first difference sequences:

1. Take the sequence $s_n = 2n^2 + 3n + 2$. Write down the first 5 elements of its first difference sequence.

2. Write down a closed form for the first difference sequence by noticing the pattern.

3. Write down the second difference sequence, which is the first difference of the first difference sequence.