
COMPARATIVE STATICS AND THE PARADIGM
OF ECONOMICS

1.1 INTRODUCTION

Students who have come this far in economics will undoubtedly have encountered the standard textbook definition of economics which goes something like, "Economics is the science which studies human behavior as a relationship between ends and scarce means which have alternative uses."⁺ This is indeed the substantive content of economics in terms of the class of phenomena generally studied. To many economists (including the author), however, the most striking aspect of economics is not the subject matter itself, but rather the conceptual framework within which the above-mentioned phenomena are analyzed. After all, sociologists and political scientists are also interested in how scarce resources are allocated and how the decisions of individuals are related to that process. What economists have in common with each other is a methodology, or paradigm, in which *all* problems are analyzed. In fact, what most economists would classify as *noneconomic* problems are precisely those problems which are incapable of being analyzed with what has come to be called the *neoclassical* or *marginalist* paradigm.

The history of science includes many paradigms or schools of thought. The ptolemaic explanation for planetary motion, in which the earth was placed at the center of the coordinate system (perhaps for theological reasons), was replaced by the copernican paradigm which moved the origin to the sun. When this was done,

⁺ Taken from Lionel Robbins' classic monograph, "An Essay on the Nature and Significance of Economic Science," p. 15, MacMillan & Co., Ltd., London, 1932.

the equations of planetary motion were so vastly simplified that the older school was soon replaced (though the Ptolemaic paradigm of classical mechanics served problems of navigation). The Newtonian paradigm of classical mechanics served admirably well in physics, and still does, in fact, in most everyday problems. For study of fundamental processes of nature, however, it has been found to be inadequate and has been replaced by the Einsteinian paradigm of relativity theory.

In economics, the classical school of Smith, Ricardo, and Marx provided explanations of the growth of productive capacity, the gains from specialization and trade (comparative advantage), and the like. One outstanding puzzle persisted: the diamond-water paradox. The classical paradigm, dependent largely on a theory of value based on inputs, was incapable of explaining why water, which is essential to life, is generally available at modest cost, while diamonds, an obvious frivolity, are expensive, even if dug up accidentally in one's backyard (considering the opportunity cost of withholding one from sale).⁴ With the advent of marginal analysis, beginning in the 1870s and continuing in later decades by Jevons, Walras, Marshall, Pareto, and others, the older paradigm was supplanted. Economic problems came to be analyzed more explicitly in terms of individual choice. Values were perceived to be determined by consumers' tastes as well as production costs, and the value placed on goods by consumers was not considered to be "intrinsic," but rather depended on the quantities of that good and other goods available.

The structure of this new paradigm was explored further by Hicks, Allen, Samuelson, and others. As this was done, the usefulness and limitations of the new paradigm became more apparent. It is with these properties that this book is concerned.

1.2 THE MARGINALIST PARADIGM

Let us consider the definition of economics in more depth. Economics, first and foremost, is an *empirical science*. *Positive* economics is concerned with questions of *fact*, which are in principle either true or false. What *ought* to be, as opposed to what *is*, is a normative study, based on the observer's value judgments. In this text we shall be concerned only with positive economics, the determination of what *is*. (For expositional ease the term *positive* will generally be dropped.) Two economists, one favoring, say, more transfers of income to the poor, and the other favoring less, should still come to the same conclusions regarding the effects of such transfers. Positive economics consists of propositions which are to be tested against facts, and either confirmed or refuted.

But what is economics, and what distinguishes it from other aspects of social science? For that matter, what is social science? *Social science is the study of human behavior*. One particular paradigm of social science, i.e., the conceptual

framework under which human behavior is studied, is known as the *theory of choice*. This is the framework which will be adopted throughout this book. Its basic postulate is that individual behavior is fundamentally characterized by individual choices, or decisions.⁴

This fundamental attribute distinguishes social science from the physical sciences. The atoms and molecular structures of physics, chemistry, biology, etc., are not perceived to possess conscious thought. They are, rather, passive adherents to the laws of nature. The choices humans make may be pleasant (e.g., whether to buy a Porsche or a Jaguar) or dismal (e.g., whether to eat navy beans or potatoes for subsistence) but the aspect of choice is asserted to be pervasive.

Decisions, i.e., choices, are a consequence of the scarcity of goods and services. Without scarcity, whatever social science might exist would be vastly different than the present variety. That goods and services are scarce is a second, though not independent postulate of the theory of choice. Scarcity is an "idea" in our minds. It is not in itself observable. However, we assert scarcity because to say that certain goods or services are *not* scarce is to say that we can all—you, me, everybody—have as much as we want of that good at any time, at zero sacrifice to us all. It is hard to imagine such goods. Even air, if it is taken to mean *fresh* air, is not free in this sense; society must in fact sacrifice consumption of other goods, through increased production costs, if the air is to be less polluted.

Scarcity, in turn, depends upon postulates about individual preferences, in particular that people prefer more goods to less. If such were not the case, then goods, though *limited* in supply, would not necessarily be *scarce*.

The fact that goods are scarce means that choices will have to be made somehow regarding both the goods to be produced in the first place and the system for rationing these final goods to consumers, each of whom would in general prefer to have more of those goods rather than less. This problem, which is often taken as the definition of economics, has many aspects. How are consumers' tastes formed, and are those tastes dependent on ("endogenous to") or independent of ("exogenous to") the allocative process? How are decisions made with regard to whether goods shall be allocated via a market process or through the political system? What system of *rules*, i.e., *property rights*, is to be used in constraining individual choices? The issues generated by the scarcity of goods involve all the social sciences. All are concerned with different aspects of the problem of choice.

We now come to the fundamental conceptualization of the determinants of choice upon which the neoclassical, or marginalist, paradigm is based. We shall assert that for a wide range of problems individual choice can be conceived to be determined by the interaction of two distinct classifications of phenomena:

1. tastes, or preferences
2. opportunities, or constraints

⁴ Of course, being different commodities with different "quantity" measurements, it is not possible to say that diamonds are *more* expensive than water.

⁴ A complicating feature, not relevant to the present discussion but also peculiar to the social sciences, is that the participants often have a vested interest in the results of the analysis.

observation, it will be impossible to tell whether it was tastes or opportunities that caused the change in consumption of bread. Confidence in the explanation based on changing opportunities will be built up only if the propositions so derived have wide application and are successful in predicting events in numerous trials. Moreover, economists (this author included) believe that these explanations based on changes in opportunities are sufficiently robust to merit serious study.

1.3 THEORIES AND REFUTABLE PROPOSITIONS

In the past several pages we have used the terms *theory*, *propositions*, and *confirm* as well as other phrases that warrant a closer look. In particular, what is a theory, and what is the role of theories in scientific explanations?

It is sometimes suggested that the way to attack any given problem is to "let the facts speak for themselves." Suppose one wanted to discover why motorists were suddenly waiting in line for gasoline, often for several hours, during the winter of 1973-1974, the so-called energy crisis. The first thing to do, perhaps, is to get some facts. Where will they be found? Perhaps the government documents section of the local university library will be useful. A problem arises. Once there, one suddenly finds oneself up to the ears in facts. The data collected by the United States federal government and other governments fill many rooms. Where should one start? Consider, perhaps, the following list of "facts."

1. Many oil-producing nations embargoed oil to the United States in the fall of 1973.
2. The gross national product of the United States rose, in money terms, by 11.5 percent from 1972 to 1973.
3. Gasoline and heating oil are petroleum distillates.
4. Wage and price controls were in effect on the oil industry during that time.
5. The average miles per gallon achieved by cars in the United States has decreased due to the growing use of antipollution devices.
6. The price of food rose dramatically in this period.
7. Rents rose during this time, but not as fast as food prices.
8. The price of tomatoes in Lincoln, Nebraska was 39 cents per pound on September 14, 1968.
9. Most of the pollution in the New York metropolitan area is due to fixed, rather than moving, sources.

The list goes on indefinitely. There are an infinite number of facts. Most readers will have already decided that, e.g., fact 8 is irrelevant; and most of the infinite number of facts that might have been listed are irrelevant. But why? How was this conclusion reached? Can fact 8 be rejected solely on the basis that *most* of us would agree to reject it? What about facts 4 and 5? There may be less than perfect agreement on the relevance of some of these facts.

Facts, by themselves, do not explain events. Without some set of axioms, propositions, etc., about the nature of the phenomena we are seeking to explain, there is simply no way in which to sort out the relevant from the irrelevant facts. The reader who summarily dismissed fact 8 as irrelevant to the events occurring during the energy crisis must have had some behavioral relations in mind which suggested that the tomato market in 1968 was not a determining factor. Such a notion, however rudimentary, is the start of a *theory*.

The Structure of Theories

A theory, in an empirical science, is a set of explanations or predictions about various objects in the real world. Theories consist of three parts:

1. A set of *assertions*, or postulates, denoted $A = \{A_1, \dots, A_n\}$, concerning the behavior of various *theoretical constructs*, i.e., idealized (perhaps mathematical) concepts, which are ultimately to be related to real world objects. These postulates are generally universal-type statements, i.e., propositions of the form: all x have the property p . Examples of such propositions in economics are the statements that "firms maximize wealth (or profits)," "consumers maximize utility," and the like. At this point, terms such as *firms*, *consumers*, *prices*, *quantities*, etc., mentioned in these behavioral assertions, or postulates, are ideas yet to be identified. They are thus referred to as *theoretical constructs*.
2. If behavioral assertions about theoretical constructs are to be useful in empirical science, these postulates must be related to real objects. The second part of a theory is therefore a set of *assumptions*, or *test conditions*, denoted $C = \{C_1, \dots, C_n\}$, under which the behavioral postulates are to be tested. These assumptions include statements to the effect that "such-and-such variable p , called the *price of bread* in the theoretical assertions, in fact corresponds to the price of bread posted at xyz supermarket on such-and-such date."

Note that we are distinguishing the terms *assertions* and *assumptions*. There has been a protracted debate in economics over the need for realism of assumptions. The confusion can be largely eliminated by clearly distinguishing the behavioral postulates of a theory (the assertions) from the specific test conditions (the assumptions) under which the theory is tested.

If the theory is to be at all useful, the assumptions, or test conditions, must be *observable*. It is impossible to tell whether a theory is performing well or badly if it is not possible to tell whether the theory is even relevant to the objects in question. The postulates A are universal statements about the behavior of abstract objects. They are not observable; therefore, debate as to their realism is irrelevant. Assumptions, on the other hand, are the link between the theoretical constructs and real objects. Assumptions *must* be *realistic*, i.e., if the theory is to be validly tested against a given set of data, the data must conform in essential ways to the theoretical constructs.

Suppose, for example, we wish to test whether a rise in the price of gasoline reduces the quantity of gasoline demanded. It will be observed that the money

price of gasoline has been rising generally since World War II and that gasoline consumption has also been rising. Does this refute the behavioral proposition that higher prices lead to less quantity demanded? Perhaps the data, specifically the assumptions about prices, are not realistic. Does the reported series of prices really reflect the intended characteristics of the theoretical construct: price of gasoline? A careful statement of the law of demand involves changes in *relative* prices, not absolute money prices, and other things, e.g., incomes and other prices, are supposed to be held fixed. When compensated by price-level changes, the *real* price of gasoline, i.e., the price of gasoline relative to other goods, has indeed been falling, thus tending to *confirm* the law of demand. But in order to test the law of demand with this datum, the assumptions about income, prices of closely related goods, etc., must also be realistic, i.e., conform to the essential aspects of the theoretical constructs.

We say *essential aspects* of the theoretical constructs because it is impossible to describe, in a finite amount of time and space, every attribute of a given real object. The importance of *realism* of assumptions is to make sure that the attributes not specified do not significantly affect the test of the theory. In the foregoing example, money prices were an *unrealistic* measure of gasoline prices; i.e., they did not contain the attributes intended by the theory. The assumptions, or test conditions, of a theory *must*, therefore, be realistic; the assertions, or behavioral postulates are never realistic because they are unobservable.

3. The third part of a theory comprises the events $E = \{E_1, \dots, E_n\}$ that are predicted by the theory. The theory says that the behavioral assertions A imply that if the test conditions C are valid (realistic), then certain events E will occur. For example, the usual postulates of consumer behavior (utility maximization with diminishing marginal rates of substitution between commodities), which we shall denote A , imply that if the test conditions C hold, where C includes decreasing relative price of gasoline with real incomes and other prices to be held fixed—that is, these assumptions are in fact *observed* to be true—then the event E , *higher gasoline consumption*, will be observed. Note that *both* the assumptions, or test conditions C and the events E must be observable. Otherwise, we can't tell whether the theory is applicable.

The logical structure of theories is thus that the assertions A imply that if C is true, then E will be true. In symbols, this is written

$$A \rightarrow (C \rightarrow E)$$

where the symbol \rightarrow means *implies*. By simple logic, the symbolic statement can also be written

$$(A \cdot C) \rightarrow E$$

That is, the postulates A and assumptions C together imply that the events E will be observed.

Refutable Propositions

We have spoken casually of *testing theories*. What is it that is being tested, and how does one go about it? In the first place, there is no way to test the postulates A directly. Suppose, to take a classic example, one wished to test whether a given firm maximized profits. How would you do it? Suppose the accountants supplied income statements for this year and past years together with the corporate balance sheets. Suppose you found that the firm made \$1 million this year. Could you infer from this that the firm made *maximum* profits? Perhaps it could have made \$2 million, or \$10 million. How would you know?

Maybe we should ask an easier question. Is the firm *minimizing* profits? Certainly not, you say. After all, it made a million dollars. Well, maybe it was in such a good business that there was simply no way to make less than a million dollars. No, you insist, if the owners of this firm were out to minimize profits, we should expect to see them giving away their goods free, hiring workers at astronomical salaries, throwing sand into the machinery, and indulging in a host of other bizarre behaviors. Precisely. The way one would *infer* that profits were being minimized would be to predict that if such behavior were present, then the given firm would engage in certain predicted events, specified in advance, such as the actions named. Since the object in question is undoubtedly a firm, i.e., the test conditions or assumptions C are realistic, and the events predicted by profit-minimization do not occur, the behavioral assertion A , that the firm minimizes profits, is refuted. *But the postulates are refutable only through making logically valid predictions about real, observable events based on those postulates, under assumed test conditions, and then discovering that the predictions are false.* The postulates are not testable in a vacuum. They can only be tested against real facts (events) under assumed, observable test conditions.

We have not, however, shown that firms maximize profits. But, we do know something. It will not be possible to determine whether firms maximize profits on the basis of whether we think that that is a sensible or achievable goal. The way to test the postulate of profit-maximization is to derive from that postulate certain behavior that should be observed under certain assumptions. Then, if the events predicted do indeed occur, we shall have evidence as to the validity of the postulate. The theory will be confirmed. But will it be *proved*? Alas, no. The nature of logic forbids us to conclude that the postulates A are true, even if C and E are known to be true. This is such a classic error it has a name. It is called the fallacy of *affirming the consequent*. If A implies B , then if B is true, one cannot conclude that A is true. For example, "If two triangles are congruent, then they are similar," is a valid proposition. However, if two triangles are known to be similar, one cannot conclude that they are also congruent, as counterexamples are easily demonstrated.

A striking example of why theories cannot be proved is presented in Fig. 1-1. The theory that the earth is round is to be tested by having an observer on the seashore note that when ships come in from afar, first the smoke from the smokestacks is visible, then the stacks, and so on, from the top of the ship on down.

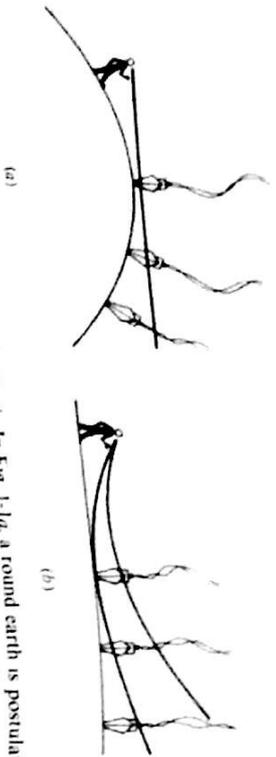


Figure 1-1 Two Theories of the Shape of the Earth. In Fig. 1-1a, a round earth is postulated. Under the assumption that light waves travel in straight lines, ships coming in from afar become visible from the top down, as they approach the shore. This is confirmed by actual observation. However, under this does not prove that the earth is round. In Fig. 1-1b, a flat earth is postulated. However, under the assumption that light waves travel in curves convex to the surface of the earth, the same events are predicted. Therefore, on the basis of this experiment alone, no conclusion can be reached concerning the shape of the earth.⁴

Panel *a* shows why this is to be expected. It does, in fact, occur every time. However, panel *b* shows that an alternative theory leads to the same events. Here, the earth is flat, but light waves travel in curves convex to the surface of the earth. The same events are predicted. There is no way, on the basis of this experiment, to determine which theory is correct. It is always possible that a new theory will be developed which will explain a given set of events. Hence, theories are in principle, as a matter of logic, unprovable. They can only be confirmed, i.e., found to be consistent with the facts. The more times a theory is confirmed, the more strongly we shall believe in its postulates, but we can never be sure that it is true.⁵

What types of theories are useful in empirical science, then? The only theories that are useful are those which might be wrong, i.e., might be refuted, but are not refuted. A theory which says that it will either rain or not rain tomorrow is no theory at all. It is incapable of being falsified, since the predicted "event" is logically true. A theory which says that if the price of gasoline rises, consumption will either rise or fall is similarly useless and uninteresting, for the same reason. The only theories which are useful are those from which *refutable hypotheses* can be inferred. The theory must assert that some event *E* will occur and, moreover, it must be possible that *E* will *not* occur. Such a proposition is, at least in principle, refutable. The facts may refute the theory; for if *E* is false, then as a matter of logic ($A \cdot C$) is false. (If nonoccurrence of the event *E* is always attributed to false or unrealistic test conditions or assumptions *C* then the theory is likewise nonrefutable.)

The paradigm of economics, therefore, in order to be useful, must consist of refutable propositions. Any other kind of statement is useless. In the various chapters of this book, we shall demonstrate how such refutable hypotheses are derived from behavioral postulates in economics.

⁴ See Irving M. Copi, "Introduction to Logic," 4th ed., Macmillan, New York, 1972.

1.4 THEORIES VERSUS MODELS: COMPARATIVE STATICS

The testing of a theory usually involves two fairly distinct processes. First, the purely logical aspects of the theory are drawn out. That is, it is shown that the behavioral postulates imply certain behavior for the variables of the theory. Then, at a later stage, the theoretical constructs are applied to real data, and the theory is tested empirically. The first stage of this analysis is what we shall be concerned with in this book. To distinguish the two phases of theorizing, we shall employ a distinction introduced by A. Papandreou and amplified by M. Bronfenbrenner.⁶ The purely logical aspect of theories will be called a *model*. A model becomes a theory when assumptions relating the theoretical constructs to real objects are added. Models are thus logical systems. They cannot be true or false empirically; rather, they are either logically valid or invalid. A theory can be false either because the underlying model is logically unsound or because the empirical facts refute the theory (or both occur).

The notion of a refutable proposition is preserved, however, even in models. A refutable proposition in a logical system means that when certain *conceptual* test conditions occur, the theoretical variables will have restricted values. Suppose that in a certain model, if a variable denoted *p*, ultimately to mean the price of some good, increases, then another variable *x* ultimately to mean the quantity of that good demanded, can validly be inferred to, say, decrease, as a matter of the logic of the model, then a refutable proposition is said to be asserted. The critical thing is that the variable *x* is to respond in a given manner, and it must be possible for *x* not to respond in that manner.

The logical simulation, usually with mathematics, of the testing of theories in economics is called the *theory of comparative statiscs*. The word *statiscs* is an unfortunate misnomer. Nothing really static is implied in the testing of theories. Recall that, in economics, theories are tested on the basis of *changes* in variables, when certain test conditions or assumptions change. The use of the term comparative statiscs refers to the absence of a prediction about the *rate* of change of variables over time, as opposed to the *direction* of change.

The testing of theories is simulated by dividing the variables into two classes:

1. Decision, or choice, variables.
2. Parameters, or variables exogenous to the model, i.e., not determined by the actions of the decision maker. The parameters represent the *test conditions* of the theory.

Let us denote the decision or choice variable (or variables) as *x*, and the

⁶ Andreas Papandreou, "Economics as a Science," J. B. Lippincott Company, Philadelphia, 1958.

⁷ Martin Bronfenbrenner, A Middlebrow Introduction to Economic Methodology, in S. Krupp (ed.), "The Structure of Economic Science," Prentice-Hall, Inc., Englewood Cliffs, N.J., 1966.

parameters of the model as α . To be useful, the theory must postulate a certain set of choices x as a function of the test conditions α

$$x = f(\alpha) \quad (1-1)$$

That is, given the behavioral postulates A, then if certain test conditions C, represented in the model by α , hold, then certain choices x will be made. Hence, x is functionally dependent on α , as denoted in equation (1-1), above.

As an empirical matter, economists will rarely, if ever, be able to test relations of the form (1-1) directly, i.e., formulate hypotheses about the actual amount of x chosen for given α . As mentioned earlier, to do this would require full knowledge of tastes as well as opportunities. The neoclassical economic paradigm is therefore based on observations of *marginal* quantities only. These marginal quantities are the responses of x to *changes* in α .

Mathematically, for "well-behaved" (differentiable) choice functions, it is the properties of the derivative of x with respect to α , or

$$\frac{dx}{d\alpha} = f'(\alpha) \quad (1-2)$$

that represents the potentially refutable hypotheses in economics. Most frequently, all that is asserted is a sign for this derivative. For example, in demand theory, prices p are exogenous, i.e., parameters, while quantities demanded x are choice variables. The law of demand asserts (under the usual qualifications) that $dx/dp < 0$. Since it is possible that $dx/dp > 0$, and since this would contradict the assertions of the model, the statement $dx/dp < 0$ is a potentially refutable hypothesis. *Comparative statics is that mathematical technique by which an economic model is investigated to determine if refutable hypotheses are forthcoming.* If not, then actual empirical testing is a waste of time, since no data could ever refute the theory.

1.5 EXAMPLES OF COMPARATIVE STATICS⁴

To illustrate the above principles, let us consider three alternative hypotheses about the behavior of firms. Specifically, suppose we were to postulate that:

1. Firms maximize profits π , where π equals total revenue minus cost.
2. Firms maximize some utility function of profits $U(\pi)$, where $U'(\pi) > 0$, so that higher profits mean higher utility. Thus, profits are desired not for their own sake, but rather for the utility they provide the firm owner.
3. Firms maximize total sales, i.e., total revenue only.

⁴ The material in this section requires some knowledge of elementary calculus techniques. The student should review parts of Chap. 2 first if these tools are unfamiliar.

By what means shall these three theories be tested and compared? It is not possible to test theories by introspection. Contemplating whether these postulates sound to us like "reasonable" behavior is not an empirically reliable test. Also, asking firm owners if they behave in these particular ways is similarly unreliable. The only way to test such postulates is to derive from them potentially refutable hypotheses and ultimately to see if actual firms conform to the predictions of the theory.

What sorts of refutable hypotheses emerge from these behavioral assertions? Among the logical implications of profit-maximization is the refutable hypothesis that if a per-unit tax is applied to a firm's output, the amount of goods offered for sale will decrease. This hypothesis is refutable because the reverse can be true. We therefore begin our first example by asserting that firms maximize profits, in order to derive this implication.

Example 1 Let $R(x)$ = total revenue function (depending on output x)

$$C(x) = \text{total cost function}$$

tx = total tax revenue collected, where the per-unit tax rate t is a parameter determined by forces beyond the firm's control

If the firm sells its output in a perfectly competitive market, i.e., it is a *price taker*, then

$$R(x) = px$$

where p is the parametrically determined market price of x . If the firm is not a perfect competitor, then p is determined, along with x , via the demand curve, and revenue is simply some function of output, $R(x)$.

In the general case, the tax rate t represents the only parameter, or test condition, of the model. The first model thus becomes

$$\text{maximize} \quad \pi(x) = R(x) - C(x) - tx \quad (1-3)$$

By simple calculus, the first-order conditions for a maximum are

$$R'(x) - C'(x) - t = 0 \quad (1-4)$$

the prime denoting first derivative.

For a maximum, sufficient second-order conditions are:

$$R'' - C'' < 0 \quad (1-5)$$

Condition (1-4) is the choice function for this firm in implicit form. It states that the firm will choose that level of output such that marginal revenue (MR) equals marginal cost (MC) plus the tax (t). If the firm is a perfect competitor,