Cass Business School
CITY UNIVERSITY LONDON

MSc Mathematical Trading and Finance
SMM609 RISK ANALYSIS AND MODELLING
Individual Coursework 2012

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PART I

General Rules

Read Carefully this part. No exceptions will be made. Penalisations will be done if these rules are not fully satisfied

RULES

This is an individual coursework. Rules as follow.

1. Group size: individual coursework.
2. Deadline: see Moodle.
3. Extensions on deadline will only be granted by the course director.
4. The coursework has to be submitted electronically via Moodle.
5. **Make sure your report is in .pdf format**. No other format will be accepted. Excel and Matlab files must be provided in the zip format.
6. Data ara available via Moodle.
7. In answering to the following questions, please state explicitily your assumptions, what data analysis has been done, your results, references you have used.
8. All figures and tables must be labeled and have a short description. All computations done in your spreadsheet or scripts must be fully revealed.
9. Usual rules apply in case of plagiarism.
10. Any questions regarding the coursework must be asked at the beginning of each lecture or just after the break. I will not respond to e-mails or personal enquiries regarding the coursework.

PART II

Things to Do

Question 1 (Marks 25)

Consider an investor that allocates her wealth in the risk free asset and in a risky asset. The portfolio return r_p is therefore

$$r_p = wr + (1-w)\, r_f,$$

where w is the weigth of the risky asset, r_f the risk-free return and r the risky asset return. Here we assume

$$r \sim \mathcal{N}\left(\mu, \sigma^2\right),$$

so that

$$r_p \sim \mathcal{N}\left(w\mu + (1-w)\, r_f, w^2\sigma^2\right).$$

The investor has expected utility that can be expressed in terms of mean and variance, i.e.

$$E\left(u\left(r_p\right)\right) = E\left(r_p\right) - \frac{\lambda}{2} V\left(r_p\right),$$

where u is the investor utility function and λ is a variance-adversion parameter.

1. [marks 5] Show that the optimal portfolio, i.e. the one that maximizes the investor expected utility, is given by

$$w^* = \frac{\mu - r_f}{\lambda\sigma^2}.$$

and discuss, given a sample of size T on the risky return, how do you estimate the optimal portfolio if you estimate the unknown parameters using the sample mean and the sample variance.

2. [marks 10] Using the properties of the sample mean and of the sample variance, see lecture on Parametric Gaussian Var,

   - compute the bias, the standard error and mean square error of your estimator (either for finite sample and for large samples),
   - find an unbiased estimator of w^*.
   - The accuracy of your estimator depends on the accuracy of the sample mean and of the (inverse) of the sample variance: Is it more important to estimate accurately the mean or the variance?

   Hint: you can use the fact that the moments of sample variance can be computed according to the formula

$$\mathbb{E}\left(\left(\hat{\sigma}^2\right)^m\right) = \left(\frac{2\sigma^2}{T}\right)^m \frac{\Gamma\left(m + \frac{T-1}{2}\right)}{\Gamma\left(\frac{T-1}{2}\right)}, m \in R. \tag{1}$$

3. [marks 10] Verify by Monte Carlo simulation that your answers at the previous point are correct (use $T = 50$ and $T = 250$, $r_f = 0.03$, $\mu = 0.07$ and $\sigma = 0.30$). **Hint**: See similar question in Exercises 4 and 5 in Homework 4. In order to simulate a chi-square random variable in Excel you have to use the inverse cdf method. In Matlab, see Homework 4.

4. [Not compulsory: marks 5] Consider the multiasset portfolio assuming that stock returns are jointly Gaussian

$$\mathbf{r} \sim \mathcal{N}(\mu, \Sigma).$$

   Here the optimal portfolio is given by

$$\mathbf{w} = \Sigma^{-1} \frac{\mu - r_f \mathbf{1}}{\lambda}.$$

Take as estimator of μ the sample mean vector having the following property:

$$\widehat{\mu} \sim \mathcal{N}\left(\mu, \frac{\Sigma}{T}\right).$$

Take as estimator of Σ the sample covariance having the following property:

$$T\hat{\Sigma} \sim \mathcal{W}(\Sigma, T-1)$$

where $\mathcal{W}(\Sigma, T)$ is the Wishart distribution with T degrees of freedom. The inverse of the covariance matrix has an inverse Wishart distribution with parameters Σ^{-1} and T. In addition, the sample mean and the sample covariance are independent. Look on Wikipedia the properties of the inverse Wishart distribution and provide an unbiased estimator of the optimal portfolio.

QUESTION 2 (MARKS 25)

Consider the dataset DATABASE_CourseWork.xls, sheet DOW. first column refers to the market index quotations. Starting in January 2004, at the beginning of each month, using previous 250 days, up to the end of your sample estimate the covariance matrix according to

- sample covariance matrix,
- one factor market model,
- shrinkage towards the market model (fix on your own the shriking factor or look on the web page of Olivier Ledoit for the relevant Matlab code),
- shrinkage towards a constant correlation matrix (fix on your own the shriking factor or look on the web page of Olivier Ledoit for the relevant Matlab code),
- ewma with $\lambda = 0.90$,

Given your estimated covariance matrix, you build the global minimum variance portfolio according to

$$\mathbf{w}^* = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}},$$

and you hold constant the portfolio composition over the following month. Repeat the procedure for each month in your sample.

1. [marks 7.5] What is the estimation procedure that provides the most stable portfolio composition over time? **Hint**: you can measure it by computing the Euclidean distance between the portfolio composition before and after you revise your estimates.
2. [marks 7.5] What is the estimation procedure that provides the most stable realized return over time?
3. [marks 5] Given your answers to the previous two points, what should be your preferred estimation procedure of the covariance matrix?
4. [marks 5] Do you think that your answer should change if you had to estimate an efficient portfolio having an expected return larger than the global minimum variance portfolio?

QUESTION 3 (MARKS 50)

Consider spot rates (continuously compounded) in the dataset DATABASE_CourseWork.xls, sheet SpotRates.
On April 29, 2011, you hold a portfolio made of:

- 1 unit of a coupon bond expiring in 2 years with yearly coupon at 2%;

- 1 unit of a coupon bond expiring in 5 years with yearly coupon at 2.5%;
- 1 unit of a coupon bond expiring in 10 years with yearly coupon at 3%;
- 1 unit of a swaption expiring in 1 years with strike 2% to enter into a swap with tenor of 1 years;
- 1 unit of a swaption expiring in 2 years with strike 2.5% to enter into a swap with tenor of 3 years;
- 1 unit of a swaption expiring in 4 years with strike 3%, to enter into a swap with tenor of 6 years;
- The notionals are always 1000 Euro. The swaptions above allow you to enter into a swap paying fixed (the strike) and receiving a floating rate. Fixed and floating payments have yearly frequency.
- Let $P(t,T)$ to be the discount factor that is related to the spot rate $y(t,T)$ by

  $$P(t,T) = e^{-y(t,T)\times(T-t)}.$$

  Coupon bonds, paying the coupon c, are priced according to the formula

  $$\left(c\sum_{i=1}^{n} P(t,T_i) + P(t,T_n)\right) \times Notional.$$

  Swaptions are priced according to the Black formula

  $$\sum_{i=1}^{n} P(t,T_i) \times (S(t)N(d_1) - KN(d_2)) \times Notional,$$

  with

  $$S(t) = \frac{P(t,T) - P(t,T_n)}{\sum_{i=1}^{n} P(t,T_i)}, d_{1,2} = \frac{\ln\left(\frac{S(t)}{K}\right) \pm \frac{1}{2}\sigma^2(T-t)}{\sqrt{\sigma^2(T-t)}},$$

  where T is the option expiry (1, 2 and 4 years), K is the swaption strike (2%, 2.5%, 3%), T_i are the payment dates of the underlying swap (for example for the 4 year swaption, $T = 4$, $T_i = 5, 6, \ldots 10$), σ is the swap rate percentage volatility ans is taken to be equal to 30%.

1. [marks 5] Estimate the current value of your portfolio;
2. [marks 5] For each component of your portfolio and thereafter for the full portfolio, compute the theta, delta and gamma assuming as risk factor parallel changes in the spot rates. Greeks can be computed by approximating partial derivatives by first differences.
3. [marks 20] Perform a PCA Analysis on percentages changes (or on first differences) of spot rates and compute the VaR of your portfolio at 90% and 99% confidence levels at different time horizons (1 day, 10 days and 20 days) retaining one, two and three principal components, by using full revaluation and theta-delta-gamma approximation. Compute the portfolio VaR via MC simulation (at least 5000 simulations, more is better) under different assumptions on the three principal components:

   - they are gaussian;
   - each of them evolves according to an independent ewma process with λ estimated according to the ML procedure;
   - each of them evolves according to an independent garch(1,1) process with parameters estimated according to the ML procedure.
   - historical simulation

   Discuss carefully your findings, with particular reference to computational issues, accuracy of your results, data fit, etc. (answer using no more than 4 A4 pages, font 11 Times, +relevant figures/tables).

4. [marks 15] Using the different simulation methods, estimate the contribution of each component to the portfolio by using the formulas we discussed in class. In particular, discuss the VaR contribution of the three swaptions. (no more than A4 pages, font 11 Times, +relevant figures/tables)

5. [marks 5] If you were a risk manager, what should be your preferred VaR methodology? Is the previous analysis conclusive or something is still missing? (no more than one A4 page, font 11 Times)

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