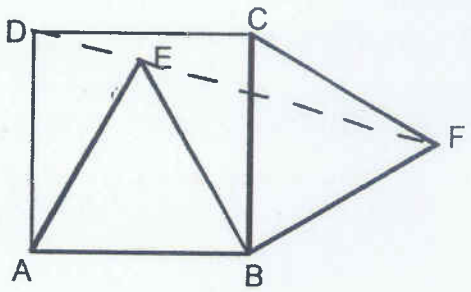
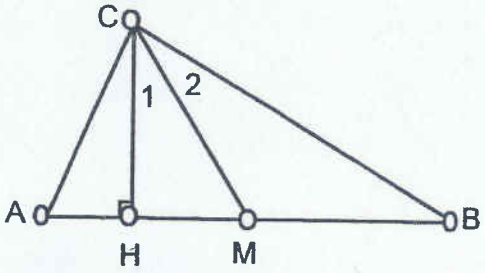
Geometry

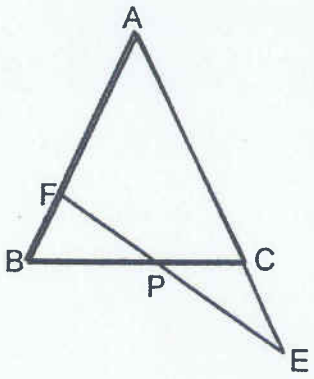
4. On sides AB and BC of square ABCD equilateral triangles ABE and BCF are constructed as shown. Prove that points D, E, and F are collinear.



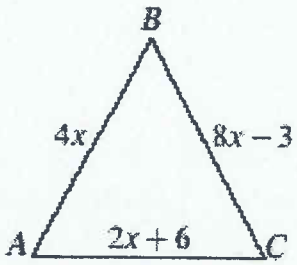
5. In right triangle ABC with the right angle at C, CH is an altitude and AC = AM. Prove that $\angle 1 = \angle 2$.



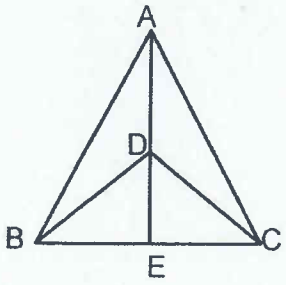
6. In isosceles triangle ABC [AB = AC] P is an arbitrary point on base BC and E is on the extension of side AC so that CE = CP. Prove that $\angle AFE = 3\angle AEF$.



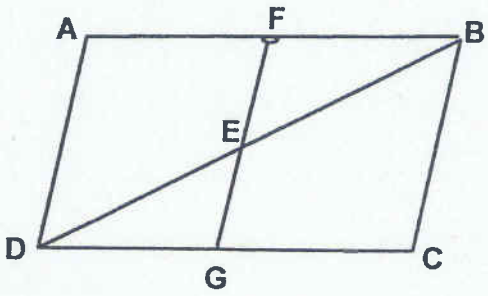
7. The lengths of the sides of a triangle are 2x + 6, 4x, and 8x – 3. For what values of x is this triangle isosceles? Justify your answer.



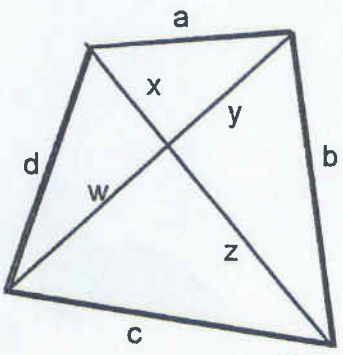
8. In the figure, triangles ABC and DBC are isosceles triangles that share base BC. Prove that line AD is perpendicular to BC.



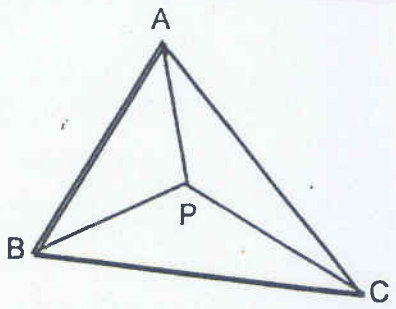
9. ABCD is a parallelogram with F is the midpoint of AB and G is the midpoint of DC. Diagonal AC intersects FG at E.
Prove that a. DE = EB b. FG || BC.



13. a. Prove that the sum of the diagonals of a convex quadrilateral is more than half its perimeter, but less than its perimeter.



b. For any point P inside triangle ABC, prove that
$\frac{1}{2} p < PA + PB + PC < p$ where p is the perimeter of the triangle.



Hint: For one of the inequalities, first prove that PA + PC < AB + BC by extending AP.