Paper Code: MATH500
Mathematical Concepts
Lecturer: Alna van der Merwe
Assignment 1
Due 5pm, Monday, 31 March 2014

Name .... **ID number**..........................

Question	Marks Possible	Marks Given
1	10	
2	10	
3	15	
4	10	
5	20	
6	10	
7	10	
8	10	
9	5	
Total	100	

Instructions:

Please attach this sheet to the front of your assignment.

The assignment must be handed in to your lecturer (Manukau students) or dropped in the Assignment Box on WT Level 1 (City students).

Answer all questions and show your working.

This is an individual assignment. The point of the assignment is for you to go through the process of discovery for yourself. Copying someone else's work will not achieve this. Plagiarism has occurred where a person effectively and without acknowledgement presents as their own work the work of others. That may include published material, such as books, newspapers, lecture notes or handouts, material from the internet or other students' written work. It also includes computer output.

The School of Computing and Mathematical Sciences regards any act of cheating including plagiarism, unauthorised collaboration and theft of another student's work most seriously. Any such act will result in a mark of zero being given for this part of the assessment and may lead to disciplinary action.

Please sign to signify that you understand what this means, and that the assignment is your own work.

Signature: ....

Question 1. [10 marks]
Let $U = \{1, 2, 3, 4, 5, 6, 7, a, b, c, d, e\}$, $A = \{2, 4, b, c\}$ and $B = \{2, 3, 6, 7, b, d, e\}$.
Find the following sets:

(a) $n(A')$

(b) $n(A \cap B')$

(c) $n(A \cup B')$

(d) $n(A' \cup B')$

List the following subsets:

(e) All subsets of A with cardinality 2

(f) All subsets of A with cardinality 3 and with b an element of the subset.

Question 2. [10 marks]
Let A, B, and C be subsets of a universal set U and suppose $n(U) = 100$, $n(A) = 31$, $n(B) = 34$, $n(C) = 35$, $n(A \cap B) = 12$, $n(A \cap C) = 10$, $n(B \cap C) = 17$, and $n(A \cap B \cap C) = 6$.

(a) Find $n(A' \cap B \cap C)$.

(b) Find $n(A \cup B \cup C)$.

Hint: Use a Venn diagram.

Question 3. [15 marks]
A fair coin is tossed three times and it is recorded whether it falls head or tail.

(a) Give the sample space S for the experiment.

(b) Consider the following events: E = a head on the first toss; F = a head on the second toss; G = a head on the third toss. Give the subset of outcomes in S that defines each of the events E, F and G.

(c) Describe the following events in terms of E, F and G and find the probabilities for the events.

- getting a head on the first toss and a head on the second toss.
- getting a head on the first toss or a head on the second toss.
- getting a tail on the first toss.
- getting at least one head in the three tosses.

(d) Are E and F mutually exclusive events? Give a reason for your answer.

Question 4. [10 marks]
Three unusual dice, A, B, and C are constructed such that die A has the numbers 3, 3, 4, 4, 8, 8; die B has the numbers 1, 1, 5, 5, 9, 9; and die C the numbers 2, 2, 6, 6, 7, 7.

(a) If dice A and B are rolled, find the probability that B beats A, that is, that the number that appears on die B is greater than the number on die A.

(b) If dice B and C are rolled, find the probability that C beats B.

(c) If dice A and C are rolled, find the probability that A beats C.

(d) Which die is the best of the three? Explain your answer.

Question 5. [20 marks]
A manufacturer of lie detectors is testing its newest design. It asks 300 people to lie deliberately and another 500 people to tell the truth. Of those who lied, the lie detector caught 200. Of those who told the truth, the lie detector accused 200 of lying. Let L describe the event that a person is a liar and N the event that the lie detector accuses a person of lying. Calculate the following probabilities and describe the events in words.

(a) $P(L \cap N)$

(b) $P(L' \cap N)$

(c) $P(L' \cup N')$

(d) $P(L|N)$

(e) $P(L'|N')$

(f) Are the events L and N mutually exclusive? Give a reason for your answer.

(g) Are the events L and N independent? Give a reason for your answer.

Question 6. [10 marks]
An experiment consists of tossing two coins. The first coin is a fair one, while the second coin is twice as likely to land with heads facing up. Find the probability that

(a) both coins land head up

(b) exactly one coin lands head up

(c) at least one coin lands head up

Question 7. [10 marks]

(a) How many 5 letter sequences are possible that use the letters m, a, t, h, s once each?

(b) How many 3 letter sequences are possible that use the letters m, a, t, h, s at most once each?

(c) How many 3 letter sets can be selected from the letters m, a, t, h, s?

(d) How many 4 letter sequences are possible that use the letters m, a, t, h, s at most once each and contains the letter m?

Question 8. [10 marks]
For a poker hand, five cards are chosen from an ordinary deck of playing cards. Find the probability to get the following hands:

(a) a hand with 1 heart, 2 diamonds, 2 clubs

(b) a hand with no face cards

(c) a hand with at least 3 queens

Question 9. [5 marks]
A bag contains two blue marbles and two red ones; two marbles are drawn at random.

(a) What is wrong with the following argument?

Because there are four possibilities – (red, red), (blue, blue), (red, blue), (blue, red) – the probability that both are red is 0.25.

(b) Find the correct probability that both marbles are red.